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Basic Concepts in Turbomachinery

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Basic Concepts in Turbomachinery

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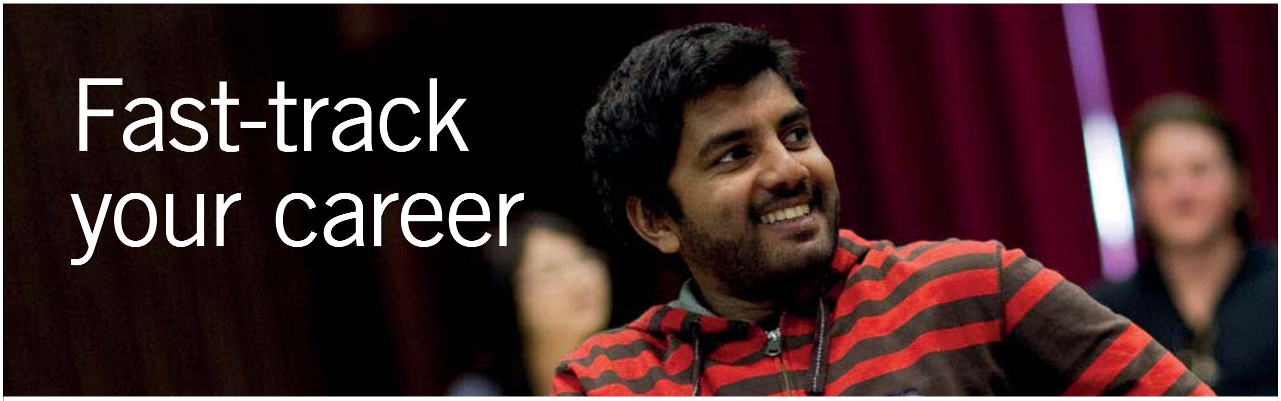

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Nomenclature

A Area

b Blade height

c Chord length

C_D Dissipation coefficient or drag coefficient

C_L Lift coefficient

C_p Specific heat capacity at constant pressure or power coefficient for wind turbines

C_v Specific heat capacity at constant volume

D Machine diameter

g Acceleration due to gravity

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h Enthalpy or static pressure head

H Total head

i Incidence angle

I Rothalpy

k Loss coefficient

K Dimensionless specific speed or loss coefficient

L Angular momentum

$\dot{m}$ Mass flow rate

N Rotation speed in revolutions per second or Dimensional specific speed

p Pressure

P Power

Q Flow rate

q Heat transfer

R Gas constant or Reaction or tip radius

r Radial coordinate

r_m Mean radius

r_t Tip radius

r_h Hub radius

s Blade pitch or entropy

t Time or blockage factor

T Torques or temperature

$\vec{U}$ Frame velocity vector

U Frame velocity magnitude

$\vec{V}$ Absolute velocity vector

V Absolute velocity magnitude

$\vec{W}$ Relative velocity vector

w Work

W Relative velocity magnitude

x Axial coordinate

z Height

α Absolute flow angle

β	Relative flow angle
γ	Ratio C_p/C_v or blade inlet angle for wind turbines
Θ	Angle made by Pelton wheel bucket
θ	Tangential coordinate
λ	Tip speed ratio for wind turbines
Π	Dimensionless parameter
η	Efficiency
ρ	Density
μ	Viscosity
σ	Thoma's parameter for cavitation
Φ	Stage loading coefficient
ψ	Flow coefficient
ω	Rotational speed

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Acknowledgements

This book is based on an introductory turbomachinery course at Durham University. This course was taught by Dr David Gregory-Smith and Professor Li He over a number of years and I am extremely grateful to them for providing a clear and lucid set of principles on which to base this work. My current colleagues at Durham Dr Rob Dominy and Dr David Sims-Williams have also provided invaluable help (even if they didn't realise it!) in preparing this work.

The book is designed to help students over some important "Threshold Concepts" in educational jargon. A threshold concept is an idea that is hard to grasp but once the idea is understood transforms the student understanding and is very hard to go back across. Within turbomachinery my view is that understanding the cascade view, velocity triangles and reaction form three threshold concepts, perhaps minor ones compared to the much bigger ideas such as "reactive power" or "opportunity cost" that are also proposed but this view has significantly influenced the production of this book. I'd therefore like to acknowledge Professor Eric Meyer for introducing me to the idea of threshold concepts.

About the Author

Grant Ingram has been a Lecturer in Fluid Mechanics at Thermodynamics at the University of Durham since 2005. He spent time working in the power generation industry on everything from large steam turbines, large and small gas turbines, pumps and hydro-electric turbines before returning to academic life to complete a PhD on turbine aerodynamics sponsored by Rolls-Royce. At the University Grant Ingram conducts research on making Turbomachinery more efficient with a particular emphasis on three dimensional design techniques for high performance turbomachinery. He also works on renewable devices work and has conducted a number of studies on small wind turbines both computationally and using experimental testing. He lectures on Thermodynamics, Turbomachinery and Fluid Mechanics at undergraduate and MSc level as well as directing short courses for industry in Thermodynamics and Turbomachinery.

Chapter 1

Introduction

This book is designed to help you understand turbomachinery. It aims to help you over some of the difficult initial concepts so that your work or study with turbomachinery will be much more fruitful. It does not tell you how to design a turbomachine but instead aims to make your other studies, lectures and textbooks which go into more depth make much more sense. For those readers not concerned with turbomachinery design it might provide all the background they need. It is based on an introductory course taught at Durham University for some years.

There are actually only three really difficult ideas in this book: understanding the cascade view (Chapter 1), velocity triangles (Chapter 2)and the concept of reaction (Chapter 6). Once you have mastered those three concepts Turbomachinery actually becomes relatively straightforward!

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1.1 How this book will help you

The book is designed to provide guidance on the basics. So if someone is presenting a velocity triangle which you do not understand or you have absolutely no idea what a stator is this book will help. Armed with this understanding you can then go on use the more complex texts effectively.

The book also contains examples which illustrate how understanding these basic concepts lead to an immediate appreciation of why machines look the way they do. So for example you will rapidly be able to see why wind turbine blades are twisted, why a the blade height in a steam turbine increases towards the low pressure end and why pump blades often point away from the direction of rotation.

The most valuable learning experience however is to actually manipulate the ideas contained in this text. A series of problems are provided at the end of each chapter with numerical answers - to fully understand the material in this book you should attempt these problems.

1.2 Things you should already know

This book is directed at readers with a basic knowledge of Fluid Mechanics and Thermodynamics. In order to make best use of the book you should have some knowledge of the steady flow energy equation, static and stagnation conditions, the perfect gas law, how to use steam tables and charts and an understanding of the boundary layer.

1.3 What is a Turbomachine?

A turbomachine is a device that exchanges energy with a fluid using continuously flowing fluid and rotating blades. Examples of these devices include aircraft engines and wind turbines.

If the device extracts energy from the fluid it is generally called a *turbine*. If the device delivers energy to the fluid it is called a *compressor, fan, blower* or *pump* depending on the fluid used and the magnitude of the change in pressure that results. Turbomachinery is the generic name for all these machines.

Somewhat confusingly the word turbine is sometimes applied to a complete engine system on an aircraft or in a power station, e.g. “a Boeing 747 is equipped with four gas turbines for thrust”. A glossary is in Appendix A on page 137 at the end of the book to help you navigate your way through the turbomachinery jargon.

Turbomachinery is essential to the operation of the modern world. Turbines are used in all significant electricity production throughout the world in steam turbine power plants, gas turbine power plants, hydro-electric power plant and wind turbines. Pumps are used to transport water around municipal water systems and in homes, pumps and turbines are also essential in the transportation of fuel oil and gas around pipe networks. Gas turbine engines are used to power all large passenger

Figure 1.1: Applications of Turbomachinery

aircraft either in the form of turbo-prop or turbo-fan engines and also through a gearbox they power all helicopter engines.

In short turbomachinery is all around you and is an area worthy of further study! Figure 1.1 shows four important applications of turbomachinery, in the top left gas turbine propulsion for aeroplanes, in the top right wind turbine power of electricity production, in the bottom left the rotor of a steam turbine for power production and a water pump is shown in the bottom right.

1.4 A Simple Turbine

There are many variants of turbine, here we describe the operation of a simple turbine so you get a feel for what is going on. An outline of a turbine is shown in Figure 1.2. From this view all we know about the device is that flow goes into it and as if by magic the shaft turns and produces a torque.

If we look at the device in an exploded view (Figure 1.3) we see that as well as a number of covers and bearings there is a row of aerodynamically shaped objects that don't move followed by a row of aerodynamically shaped objects that provide the torque to the shaft.

The objects are known various as blades, buckets, nozzles, aerofoils or airfoils. In this book we will generally refer to them as blades. The row of stationary blades is known as a stator and the row of rotating blades connected to the output shaft is known as the rotor.

The basic mechanism of operation is as follows (Figure 1.4):

1. the fluid flows directly into the device in an axial direction (in line with the machine)
2. the stator blades turn the flow so that it is lined up with the turbine blades
3. the turbine blades turn the flow back towards the axial direction and turn the output shaft.

The key point is that the power extraction from the fluid arises from *turning the flow*. More complex turbines use more than one row of rotors and stators, but all work on the same essential principle. A question often asked at this point is that since all the power comes from the rotor can you do without a stator? The answer is yes! Wind turbines extract power from fluid with the need for a stator. However for flows with much larger energy densities such as those in aircraft engines adding a stator allows you to get much more energy out of the subsequent stator row - the reason for this is found in Chapter 5.

1.5 The Cascade View

There are two key views of turbomachinery used throughout this book (and in turbomachinery design in general). These are the *cascade view* and the *meridional view*.

The cascade view arises from looking at the stator and rotor of the simple turbine shown earlier (Top half of Figure 1.5) if you look closely at the topmost part of the turbine you can see the blades of the stator and rotor outlined in plan view. This is highlighted by a red box. You can actually do this for any rotor/stator blade combination around the circumference of the turbine. The fact that you can do this for every blade suggests that the plan view may be an excellent way of analysing the performance of the machine.

The 2D cascade view of the simple turbine is shown in the lower half of Figure 1.5. The cascade view with a single stator and rotor blade is highlighted with a red box. The relation between the 2D cascade view and the 3D real turbine should be obvious. The rest of the cascade view is made up of plan views of the other stator and rotor blade combinations. When looking directly down onto the red box in the the 3D view of the turbine the movement of the rotor blade appears to be simply from left

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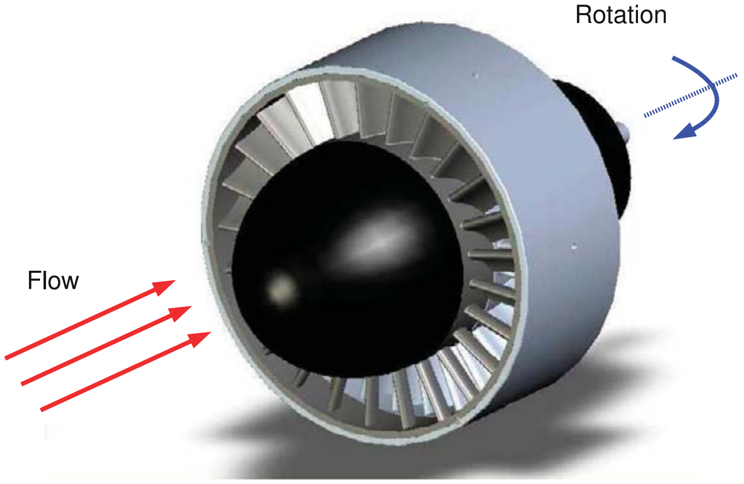
Rotation
Flow

Figure 1.2: A Simple Turbine

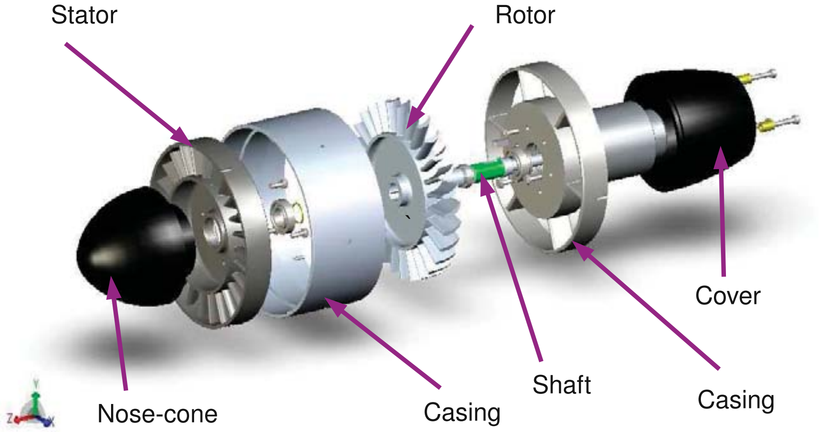
Stator
Rotor
Cover
Shaft
Casing
Nose-cone
Casing

Figure 1.3: A Simple Turbine: Exploded View

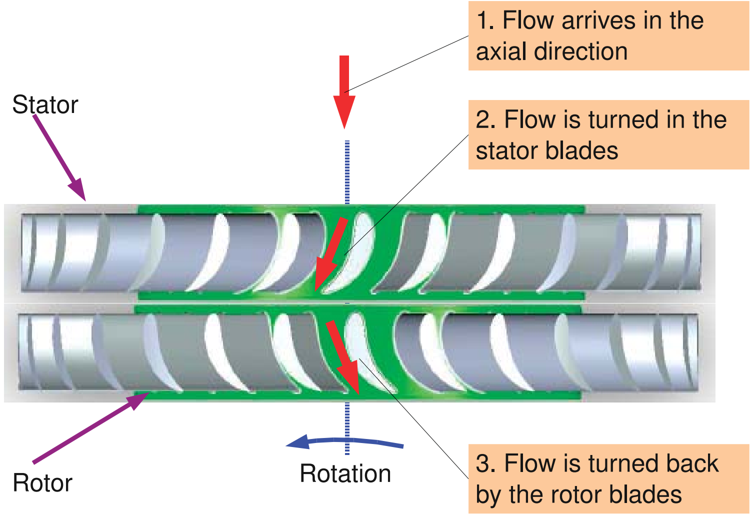
1. Flow arrives in the axial direction
2. Flow is turned in the stator blades
Stator
Rotor
Rotation
3. Flow is turned back by the rotor blades

Figure 1.4: Simple Turbine Operation

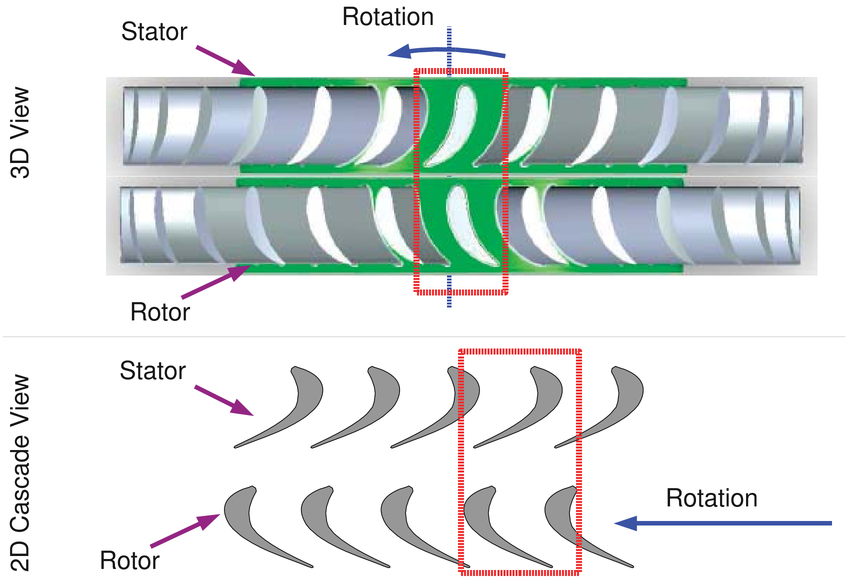
3D View
Stator
Rotation
Rotor
2D Cascade View
Stator
Rotation
Rotor

Figure 1.5: Cascade View

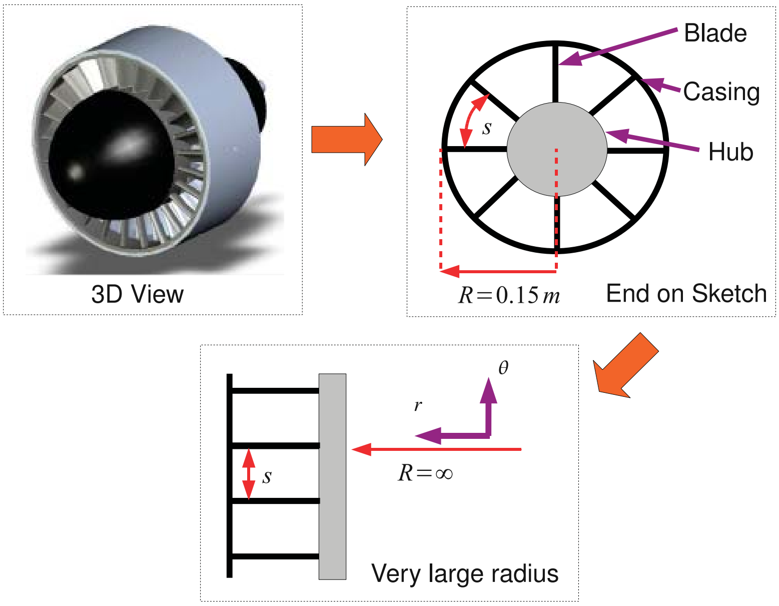

Figure 1.6: The Cascade View as a Large Radius Machine

to right. So in the cascade view the rotary motion in the 3D model becomes 2D linear motion in the cascade view.

We can then analyse how the turbine blades influence the flow by looking at this 2D cascade view, since the cascade view is the same for every blade passage around the circumference of the turbine. Although we have completed this for the top of the turbine we can repeat the exercise at any radius from the hub of the machine to the tip.

An alternative way of looking at the cascade view is to say that we are examining an infinite radius machine. Consider Figure 1.6 which contains three views, the first is a 3D view of a simple turbine, the second shows a sketch of the turbine as viewed from upstream with the blades and hub shown in schematic form. To form the cascade view we can approximate the real turbine rotating at speed ω with a tip radius $R = 0.15m$ and a spacing between the blades of s with a machine with an infinite radius and the same blade spacing (or pitch) of s. The rotation of the machine ω is replaced by a linear motion of magnitude ωR where R is the radius of the original machine.

The actual cascade view involves looking down from the casing to the hub so you get a plan view of the blades. Note that in the real machine the pitch s gets larger with larger radius r so the cascade view only accurately represents the machine at a single radius. For machines with very large changes of radius such as wind turbines we can draw a number of cascade views at different radii.

The cascade has two “analysis stations” associated with it at inlet and outlet. A consequence of the cascade view is the properties of the fluid (pressure, temperature etc) going through the machine are assumed constant in the tangential direction since there is no change in geometry or flow between one blade and the next in that direction. In the real machine this assumption represents properties being constant around the circumference of the machine so that a single value describes the fluid state around the whole machine. Analysis stations can also be applied to parts of the turbomachine that don’t always have rotational symmetry such as the inlet or the exit pipe - what is assumed there is that a single value accurately represents the flow in the inlet or the exit.

1.6 The Meridional View

The meridional view is much more straightforward than the cascade view and is illustrated in Figure 1.7. On the left of Figure 1.7 is the familiar 3D view of our simple turbine. For the meridional view instead of looking at the tip of the blade this time we take a side on view of the whole turbine and look at a cross section of the machine at the hub and tip radius. This is highlighted by a red box. On the right of Figure 1.7 is the actual meridional view which shows the stator followed by the rotor in cross section. The actual machine radius r is usually very large compared to the blade height b and so the axis of rotation is not always shown in the meridional view.

1.7 Assumptions used in the book

It is easy to see how the real turbomachinery flow field is three dimensional and unsteady now that the complex geometry of machine has been shown. In addition the flow is compressible so density changes have to be accounted for. However to introduce the basic concepts we can dispense with a great deal of this complexity by making a number of assumptions about the flow field.

1. The flow is symmetric in the circumferential direction. There is no variation in the flow from one side of the blades to another

2. We consider a mean flow (technically called a stream surface) between the hub and casing. This is reasonable for short blades, for longer blades the "trick" is to repeat the calculation at a number of radii.

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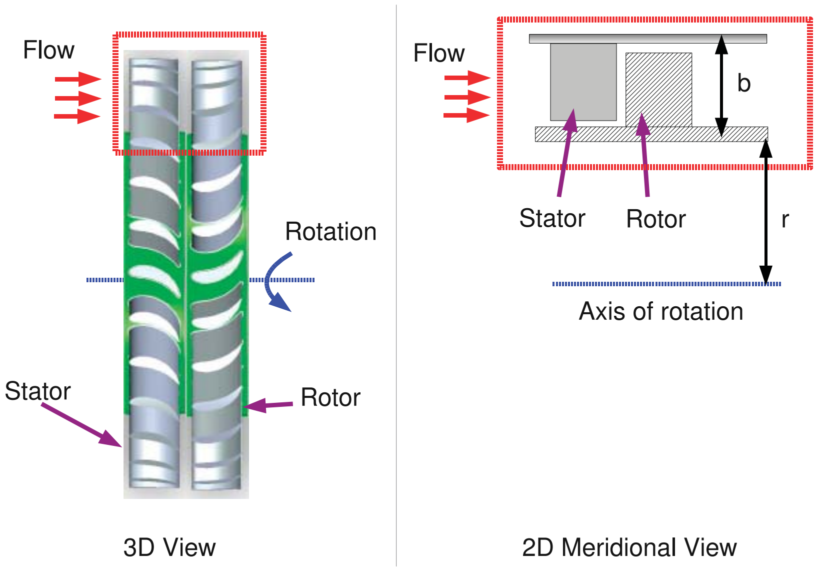

3D View

2D Meridional View

Figure 1.7: Meridional View

3. Flow is steady. Although state of the art blade design requires a consideration of unsteady flow most of the turbomachinery in use today has been designed with this steady flow assumption.
4. Flow follows the blade exactly. There is no deviation between the direction that the blades are pointing and the direction that the fluid travels in. (In turbomachinery jargon: the flow follows the metal angle of the blades)

These assumptions may seem quite limiting but most of them are used in the preliminary design of all turbomachinery in use today so actually get you a surprisingly long way!

1.8 Problems

1. Explain why a bicycle pump is not classified as a turbomachine.
2. Sketch the cascade and meridional views for a horizontal axis wind turbine such as the one in the top right of Figure 1.1.

Chapter 2

Relative and Absolute Motion

One of the key concepts in turbomachinery is understanding how the flow appears from the point of view of components that are rotating compared to those that are stationary. Once this is understood this the shape of turbomachinery becomes much easier to understand! Viewing flows from the point of view of a rotating component is known as being in the *relative frame of reference* and viewing flows from the point of view of a stationary observer is called being in the *absolute frame of reference*.

We start therefore with a simple explanation of relative and absolute motion before ending this Chapter with a discussion of how this relates to turbomachines.

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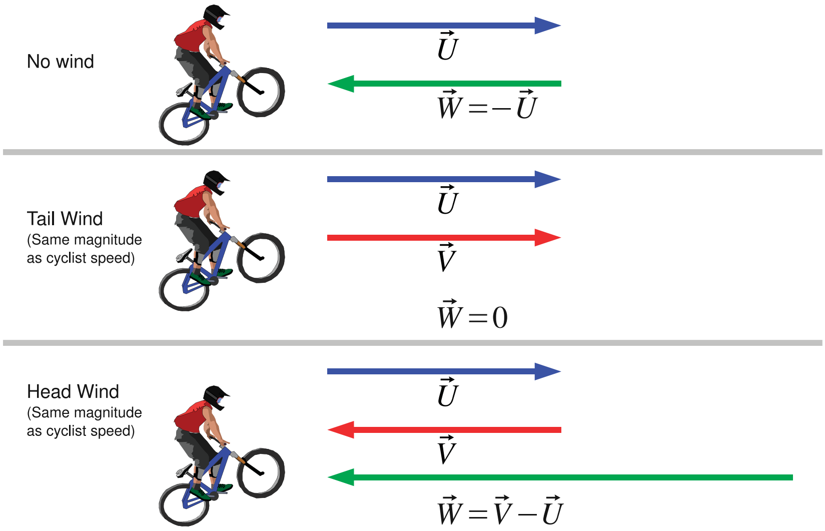


Figure 2.1: Relative and Absolute Velocities for a Cyclist

2.1 1D Motion

Consider the everyday activity of riding a bicycle with three cases one where there is no wind, the second with a tail wind and a third with a head wind. This is shown in Figure 2.1. The velocity of the bicycle we shall label $\vec{U}$ and call it the "frame velocity", the velocity of the wind we label $\vec{V}$ and call this the "absolute velocity". Clearly the absolute velocity $\vec{V}$ is the velocity that will be experienced by an observer watching the cyclist. The wind velocity experienced by the cyclist is called the "relative velocity" and given the symbol $\vec{W}$.

The first case shown at the top of Figure 2.1 shows the simplest case, if there is no wind the observer watching the cyclist will experience no wind and the cyclist will experience a relative velocity that is equal and opposite to that of the speed at which he or she is cycling. So the relative velocity $\vec{W} = -\vec{U}$.

The second case concerns a tail wind that is roughly equal in magnitude to the speed of the bicycle $\vec{U}$. This is shown in the middle of Figure 2.1. In this case a stationary observer would experience the wind velocity but since the cyclist is moving at the same speed as the air the relative velocity $\vec{W}$ will be around zero and the cyclist will experience no wind.

The third case concerns a head wind that is again roughly equal to the velocity $\vec{U}$ of the bicycle in magnitude but not in direction. This is shown at the bottom of Figure 2.1. A stationary observer would experience the same wind velocity as in the second case but in a different direction. The cyclist however has a very different experience. The relative velocity is made up of their own speed $-\vec{U}$ (that of the first case) added to that of the oncoming wind $\vec{V}$. By inspection we can see that $\vec{W} = \vec{V} - \vec{U}$. Since $\vec{V}$ is negative the cyclist now has to work much harder to maintain the same forward speed.

This suggests a generalisation of the relationship between relative and absolute velocity:

$$\vec{V} = \vec{U} + \vec{W} \tag{2.1}$$

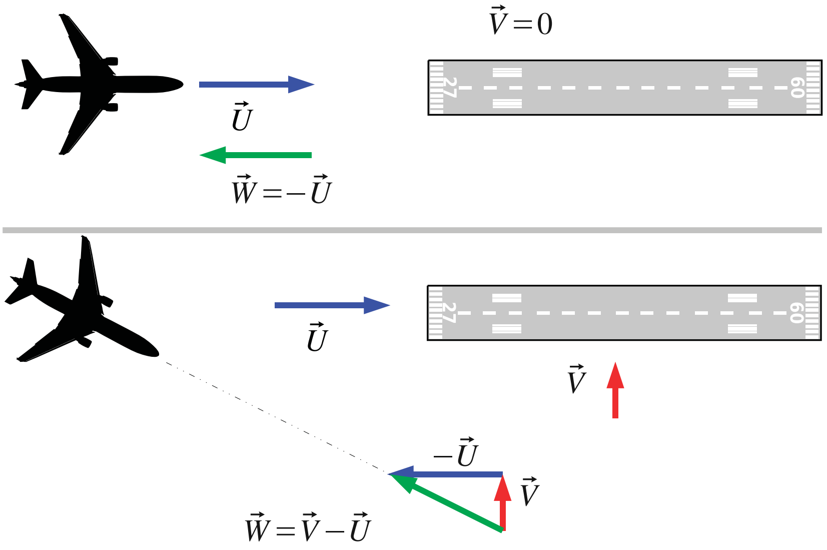


Figure 2.2: Velocity Triangles for an Aircraft Landing

Or in words absolute velocity is the vector sum of the frame velocity and the relative velocity. A trivial rearrangement returns us to the relationship seen in Figure 2.1.

$$\vec{W} = \vec{U} - \vec{V} \tag{2.2}$$

2.2 2D Motion

We will apply our new found key rule (Equation 2.1) to one other non-turbomachinery situation to illustrate how it works. This situation is one where the motion is in two dimensions. Consider the plan view of a aircraft and a runway in Figure 2.2. In the first situation (top of Figure 2.2) there is no atmospheric wind $\vec{V} = 0$ and so the aircraft simple lines up with the runway and lands.

The second situation (lower part of Figure 2.2) is where there is a substantial cross-wind, in this case imagine that the wind is entirely perpendicular to the runway. What relative velocity ($\vec{W}$) does the aircraft have to fly at to ensure that the movement of the aircraft (the frame velocity $\vec{U}$) results in the aircraft arriving on the centre-line of the runway?

The frame velocity we know is given by the desired path of the aircraft, that is directly towards the runway and the absolute velocity is given the atmospheric conditions. The relative velocity is given mathematically by the application of our key rule, Equation 2.2. But what if we wanted to sketch out the vector? This enables us to understand the direction the aeroplane should be facing.

To do this we need to use a tool known as a *velocity triangle* one of the fundamental tools of turbomachinery analysis. First we review some very basic vector addition and subtraction rules, shown in Figure 2.3.

- To add two vectors $\vec{A} + \vec{B}$ graphically: place them nose to tail and the result is given by movement from the tail of the first to the nose of the second.

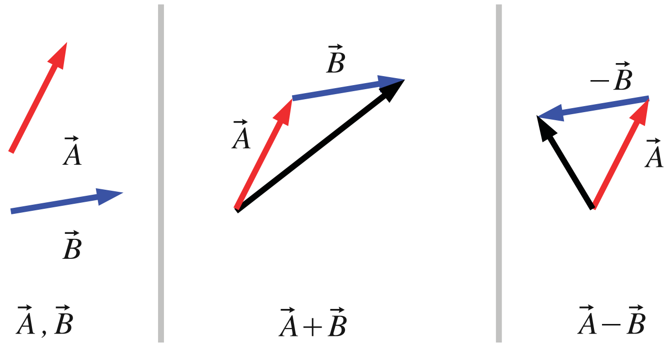


Figure 2.3: Graphical Addition and Subtraction of Vectors

- To subtract two vectors $\vec{A}-\vec{B}$ graphically: reverse the direction of $\vec{B}$ then proceed with addition of vectors as before.

To apply this to the example of our aircraft we apply the key rule and our knowledge of how to put vectors together to end up with the required relative velocity. This is shown in the lower portion of Figure 2.2, first the frame velocity $\vec{U}$ is reversed in direction to form $-\vec{U}$, this is then added to $\vec{V}$ by putting them nose to tail. We then draw the line between the start of the vector $\vec{V}$ and the end of the vector $-\vec{U}$ which gives the relative velocity $\vec{W}$.

This explains why aircraft landing in cross-winds often have to approach the runway at an angle. If you have an active web connection there are some spectacular examples of this on YouTube: `http://uk.youtube.com/watch?v=GHrLB_mlir4`

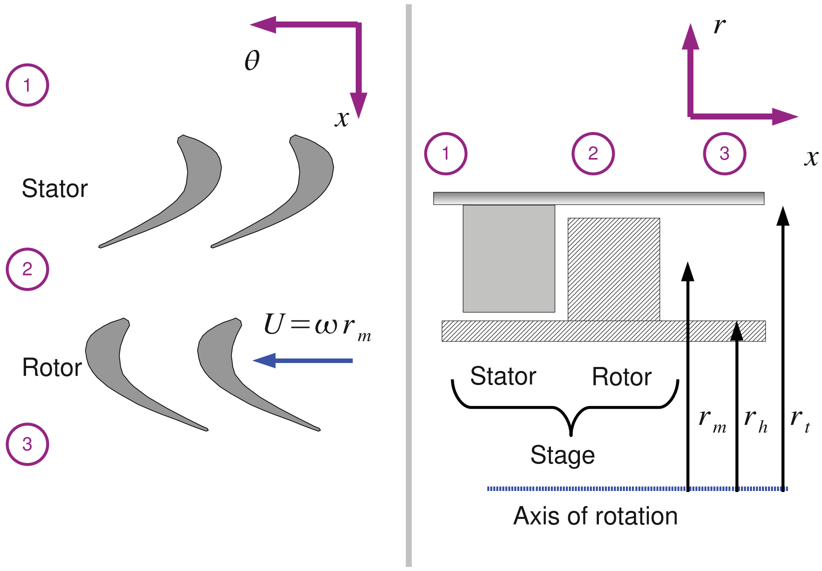
Note that we formed the relative vector $\vec{W}$ by drawing $\vec{V}$ then $-\vec{U}$ but we would end up with the same result if we drew the triangle with $-\vec{U}$ then $\vec{V}$.

All this may seem obvious but it is vitally important before we move onto turbomachinery that you are confident in how to draw a 2D vector and how to add and subtract vectors graphically.

2.3 Velocity Triangles in Turbomachinery

In this book we consider a Cartesian coordinate system consisting of an axial x, radial r and tangential θ set of coordinates. The velocity of the frame of motion is denoted by $\vec{U}$, velocities in the frame of motion are denoted with $\vec{W}$ and absolute velocities are denoted with $\vec{V}$. Consider a turbine consisting of a stator and a rotor, the cascade and meridional views are shown in Figure 2.4 along with the coordinate system.

There are three points that are of interest to us entry to the stator, the gap between the stator and the rotor and exit from the rotor, these are labelled 1,2 and 3 respectively in Figure 2.4. The


θ
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Rotor
③
U=ωr_m
r
① ② ③
x
Stator
Rotor
Stage
r_m
r_h
r_t
Axis of rotation

Figure 2.4: Cascade and Meridional Views of a Turbine Stage

combination of rotor and stator is called a “stage” in turbomachinery jargon. These points are the analysis stations referred to in Chapter 1.

At point 1 we have an incoming velocity but as the stator is not moving there is no relative motion between the incoming flow and the stator so there is no velocity triangle to draw at this point.

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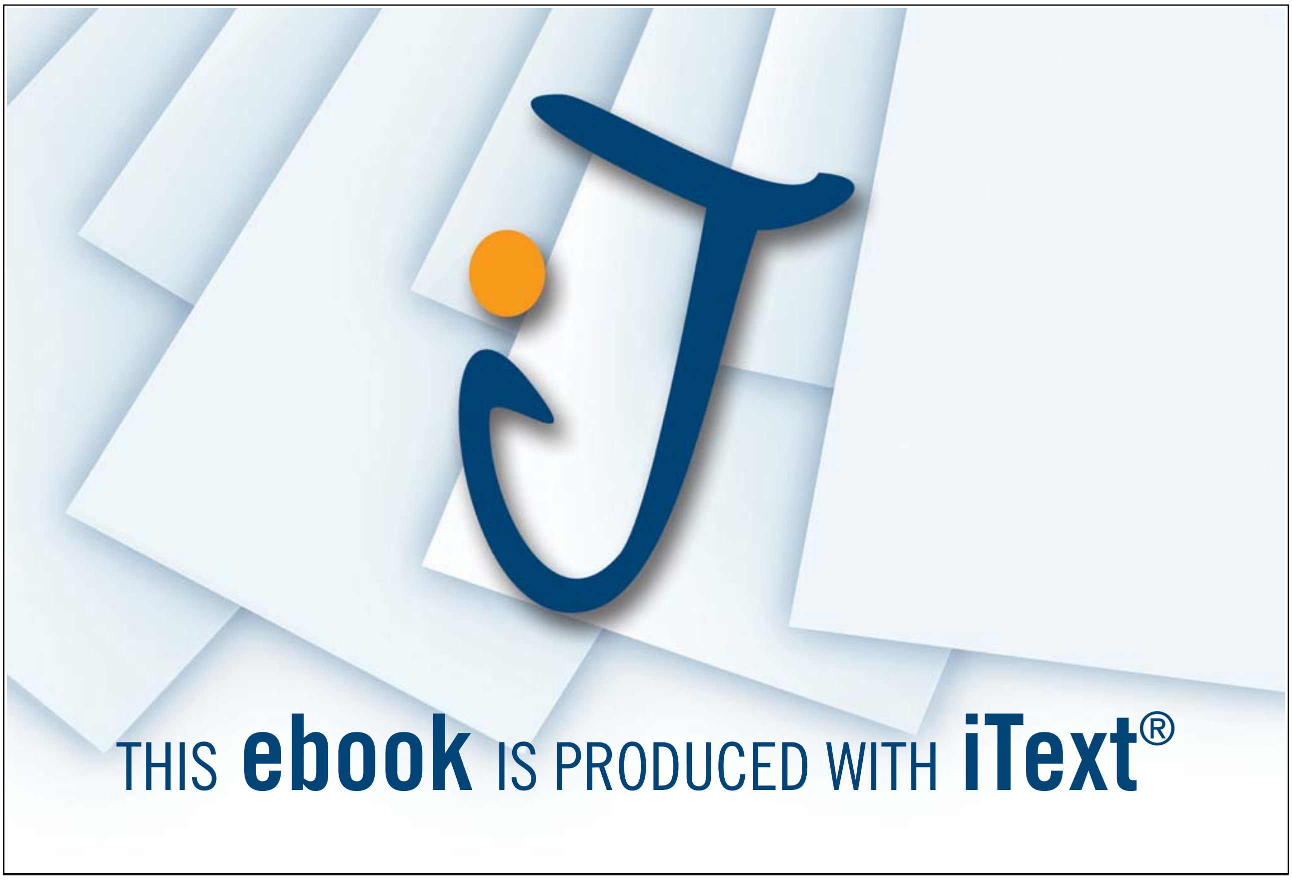
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At point 2 the flow leaves the stator and enters the rotor. Here there are two frames of reference, the flow viewed from the point of view of the stator and the point of view from the moving rotor. A velocity triangle can be drawn here. The rotor in the cascade view is moving with a linear (tangential) velocity of magnitude ωr_m where ω is the rotational speed of the machine and r_m is the mean radius of the blades.

At point 3 the flow leaves the rotor and exits the stage. Again there are two frames of reference, or points of view for the flow. That found by viewing from the moving rotor and that found by viewing from outside the rotor where there is no motion.

We can now draw the velocity triangles for point 2 and point 3 in the stage, this is shown in Figure 2.5. The methodology for this is as follows:

1. Draw the flow that you know
2. Draw the blade speed
3. Close the triangle with the remaining vector
4. Check that the key rule applies: $\vec{V} = \vec{U} + \vec{W}$

This methodology is important and we will refer to elsewhere as the “four step rule”.

So for station 2 the flow that we know is the absolute velocity at exit from the stator, $\vec{V}$. This is the flow that we know, recall from Chapter 1 that the flow follows the metal angle of the blades so if with a sketch of the stator the absolute velocity may be drawn directly. To get the velocity triangle draw the absolute velocity vector $\vec{V}$, draw the blade speed $\vec{U}$ and then close the triangle with the relative velocity $\vec{W}$. The result is in Figure 2.5. The final (and vital!) step is to check that the correct triangle has been obtained by following the blade speed and relative velocity vectors. If we end up in the same place as if we had followed the absolute velocity vector the triangle is correct.

For station 3 the flow that we know is the relative velocity at exit from the rotor, $\vec{W}$ again this is because the flow follows precisely the path of the blades and since the rotor blades are moving the flow that we sketch on the rotor must be the relative and not the absolute velocity. Having drawn $\vec{W}$ we draw the blade speed $\vec{U}$ and then close the triangle this time with the absolute velocity $\vec{V}$. Again the final vital step is to check that the we have the correct triangle by making sure the key rule applies. The correct velocity triangle is shown at the bottom right of Figure 2.5.

Once we have drawn the velocity triangles we can then carry out a series of calculations on the fluid going through the turbomachine and the end result might indicate that our sketch is not entirely accurate, i.e. the blade speed is much greater than what we have drawn in Figure 2.5 - this does not matter! If accurate velocity triangles are required they can always be drawn again once the calculations are complete.

2.4 Velocity Components

For each velocity in the cascade view we can decompose into axial and tangential components and can also express each vector as a magnitude and direction. Axial components are denoted with the subscript x and tangential components are denoted with subscript θ. Angles can be measured in a number of directions but in this book the axial direction is chosen. The angle made by the absolute

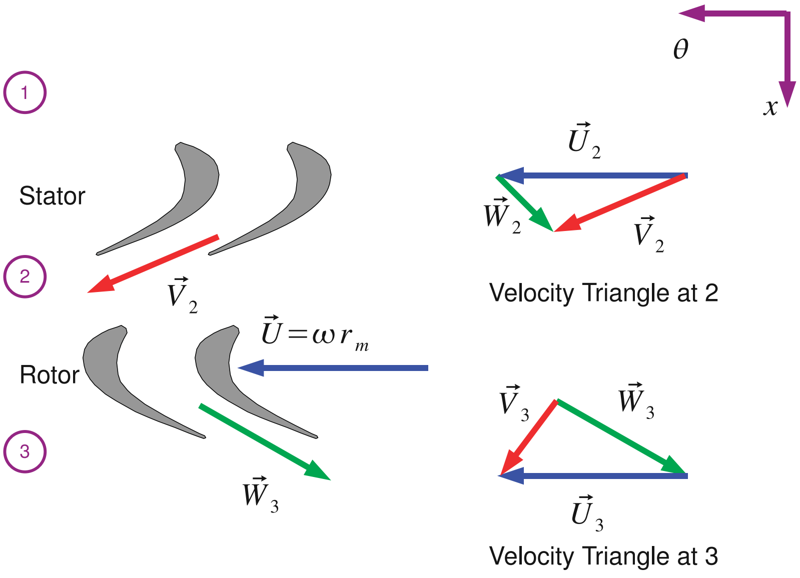
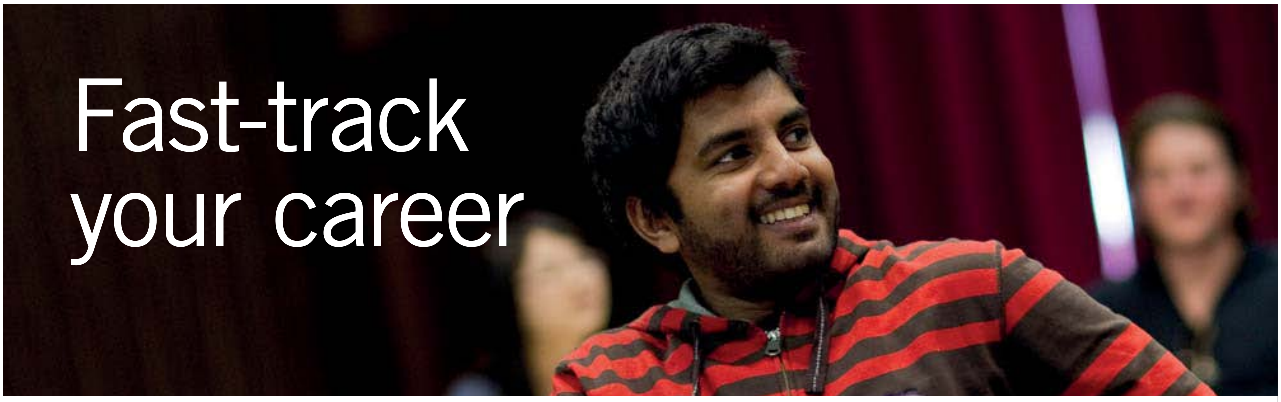


Figure 2.5: Velocity Triangles for a Turbine Stage

velocity with the axial direction is called α and the angle the relative velocity makes with the axial direction is called β. Angles are positive in the direction of rotation. Therefore velocities can be specified as a vector $\vec{V}$ or a magnitude and angle, V and α.

The one complication to this is that in this book we also deal with machines where the flow has a significant radial component - in that case we draw velocity triangles in the radial (subscript r) and



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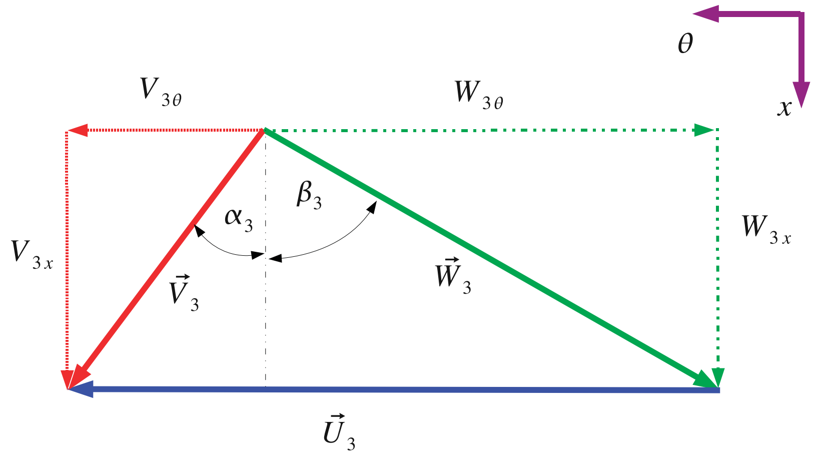

Figure 2.6: Velocity Triangles at Station 3 of a Turbine

tangential plane and angles are measured from the radial direction. This will become clearer when radial and centrifugal machines are explained in Chapter 4.

The velocity triangle at station 3 in Figure 2.5 is shown in Figure 2.6 with the various components labelled, in order to indicate that we are dealing with station 3 a subscript 3 is added to all the symbols. The relative and absolute flow angles α and β are also shown.

From basic trigonometry the follow relationships apply for any station in a turbomachine.

$$V^2 = V_\theta^2 + V_x^2 \tag{2.3}$$
$$V_x = V\cos\alpha \tag{2.4}$$
$$V_\theta = V\sin\alpha \tag{2.5}$$
$$V_\theta = V_x\tan\alpha \tag{2.6}$$
$$W^2 = W_\theta^2 + W_x^2 \tag{2.7}$$
$$W_x = W\cos\beta \tag{2.8}$$
$$W_\theta = W\sin\beta \tag{2.9}$$
$$W_\theta = W_x\tan\beta \tag{2.10}$$

Aside from trigonometry we can also work out that $W_x = V_x$ for all turbomachinery. The reason for that is if we look the basic geometry of a turbomachine such as that shown in Figure 1.4 we see that there is no motion of the machine components in the axial direction. That is the stator and the rotor remain the same distance apart when the machine is operating. The only time we would have movement between the stator and the rotor is if the device had suffered some sort of catastrophic failure - there is no normal operating procedure where the gap between the rotor and stator would increase!

Example Consider an Office Desk Fan. It rotates at 200 rpm and has a diameter of 30 cm. Air enters the fan at 3 m/s, parallel to the axis of rotation. Calculate the relative velocity ($\vec{W}$) at the tip of the fan.

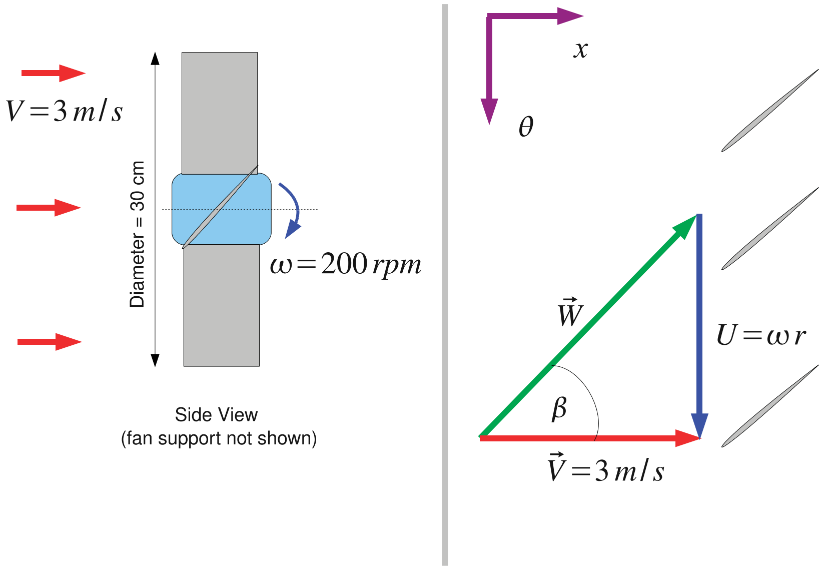

Figure 2.7: Velocity Triangles for a Desk Fan

Solution The information given is the absolute velocity $\vec{V}$ which is the air entering the fan. The plan is to work out the frame velocity and thus determine the relative velocity.

To do this sketch the fan and velocity triangle in Figure 2.7. A sketch of the desk fan is on the left hand side of Figure 2.7 and the cascade view and the velocity triangles are shown on the right. It is very good practise to sketch the object that you are trying to do calculations on to determine the overall layouts - even if (as in Figure 2.7!) the sketch is only approximate.

The frame velocity $\vec{U}$ we can obtain from the rotational speed and the radius:

$$U = \omega r = 200 \times \frac{2\pi}{60} \times \frac{0.3}{2} = 3.14\, m/s$$

Trigonometry gives the magnitude of the relative velocity:

$$W = \sqrt{V^2 + W^2} = \sqrt{3^3 + 3.14^2} = 4.34\, m/s$$

and the angle:

$$V \tan\beta = -U \implies \beta = \tan^{-1}\left(\frac{-U}{V}\right) = \tan^{-1}\left(\frac{-3.14}{3}\right) = -46.3^\circ$$

β is negative as the angle is opposite to the direction of rotation. Many students write $V \tan\beta = U$ which gets the correct magnitude but a careful inspection of Figure 2.7 will reveal that this has the wrong sign.

2.5 Problems

1. An aeroplane approaches a runway at 77 m/s with a crosswind of 15 m/s. What angle does the aeroplane have to face into the wind to travel directly towards the runway? *Answer:* 11°

2. An office desk fan rotates at 200 rpm. Air enters the fan at 3 m/s, parallel to the axis of rotation. Calculate the relative velocity ($\vec{W}$) at the hub of the fan if the hub diameter is 10 cm. *Answer:* 3.18 m/s, $-19.2°$

3. The flow at exit from a turbine stator row has a velocity of 100 m/s at an angle (α_2) of $70°$ to the axial direction. Calculate the tangential and axial velocity components. The rotor row is moving with a velocity of 50 m/s. Calculate the velocity magnitude relative to the rotor blades at inlet and the relative inlet flow angle (β_2). At exit from the rotor row the relative flow angle (β_3) is $-60°$. Assuming that the axial velocity is constant across the row, what is the absolute exit velocity magnitude and direction? *Answer:* 94.0 m/s, 34.2 m/s; 55.7 m/s, $52.1°$; 35.4 m/s, $-15.1°$

4. For the turbine above, assuming that the relative flow at exit from the rotor row is unchanged, calculate the blade speed that would give absolute axial flow at exit (i.e. no swirl) *Answer:* 59.2 m/s

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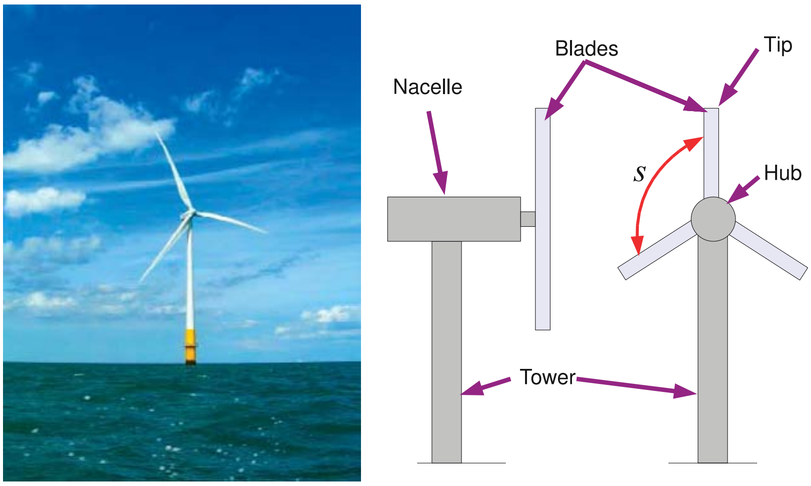
Chapter 3

Simple Analysis of Wind Turbines

This chapter provides an immediate application of the principles of relative motion by using the horizontal axis wind turbine as an example. Such a wind turbine is shown in Figure 3.1 as can be see from Figure 3.1 the blades are far apart so the influence between them is very small. In turbomachinery jargon the pitch s is very large. The interactions between the different blades can be ignored in a simple treatment. The wind turbine is one of the only examples in turbomachinery where each blade can be considered in isolation and this is one of the reasons that a simple analysis is easy.

Consider one of the turbine blades shown in Figure 3.1, the speed of each of the three blades will be the same and assuming that the wind does not vary over the area of the machine, an analysis need only be conducted on a single blade and multiplied as necessary. The interaction of the blades and the tower is ignored but since this occupies a small fraction of the 360° rotation of the turbine this



Real Wind Turbine Side View Front View

Figure 3.1: Wind Turbine Picture and Sketch

will not influence the analysis greatly.

Consider what happens if an observer was positioned on the turbine blade about half way along the span and a virtual cut made through the blade. If the observer looked towards the hub of the machine, they would see an aerofoil profile rotating around the hub. Since the radius of the machine is large the rotational motion of the turbine blade can be approximated by a linear motion - in much the same way as the earth is round but looks flat as the radius is very large. Such a view of a turbine blade is shown in Figure 3.2. The rotational motion of speed ω is translated to a linear motion of ωr the tangential velocity of the rotating blade. If the turbine is facing into the wind the incoming wind $\vec{V}$ will be perpendicular to the rotating blade.

The velocity triangle for the wind turbine blade can then be drawn according to the four step procedure and is shown on the left hand side of Figure 3.2. The flow that is known is the incoming wind velocity $\vec{V}$ which is in the absolute frame of reference. The blade speed, $\vec{U}$ is then drawn and the triangle is closed by the relative velocity $\vec{W}$. The triangle can then be checked by following the relative velocity vector and the blade speed to ensure that the same point is arrived at if $\vec{V}$ alone was followed.

The blade therefore experiences a velocity of magnitude W and angle β which will produce a force on the blade.

Recall that from basic mechanics that a force in two dimensions can be resolved into two perpendicular components of any orientation. Two particularly convenient directions are found to be perpendicular and parallel to the incoming flow. The force perpendicular to the incoming flow is known as the lift force L and the force parallel to the incoming flow is known as the drag force D. The principle reason that these are useful directions is that a large body of data on aerofoil performance is available in this form.

For the wind turbine to produce a useful output a force in the tangential direction must be produced, so the lift and drag forces must be resolved into the tangential direction to give a tangential component of the force F_θ. It is also possible to determine the axial force F_x on the blades which is

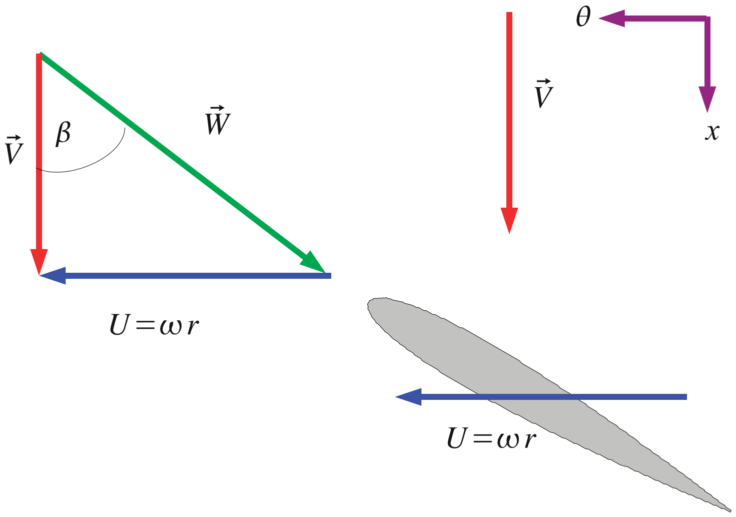

$\vec{V}$
β
$\vec{W}$
$U = \omega r$
$\vec{V}$
θ
x
$U = \omega r$

Figure 3.2: Wind Turbine Blade and Velocity Triangle

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The operating point was obtained by examining the lift as 85% of maximum where $C_L = 1.1$ from Figure 3.5 the incidence i at this point is around $10°$. Using equation 3.5 determine the blade angle:

$$|\gamma| = |i| + |\beta| = 10 + 75.1 = 85.1°$$

From Figure 3.5 we can see that the aerofoil stalls at an incidence of around $14°$ which means the relative flow angle at the stall condition is given by:

$$|\beta| = 85.1 - 14 = 71.1°$$

The velocity triangle in this example is the same as that used in the previous example so from Figure 3.2 we can see that β should be negative so for stall $\beta = -71.1°$. Using the velocity triangle gives a relationship for V:

$$\beta = \tan^{-1}\left(\frac{-U}{V}\right) \implies V = \frac{-U}{\tan\beta} = \frac{-37.7}{\tan(-71.1)} = 12.9\, m/s$$

This example illustrates why long turbine blades are *twisted*, as the radius changes so does the blade speed so to maintain the same incidence the blade has to be twisted.

Example If the blade hub section in the example above at a radius of 1.5m is designed to stall at the same wind condition, what would be the local blade angle?

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Solution So at the hub of the wind turbine the blade angle γ must be set to stall at $V = 12.9\, m/s$. Since the radius of our analysis has changed so will the blade speed:

$$U = \omega r = 1 \times 2\pi \times 1.5 = 9.4\, m/s$$

From the velocity triangle (Figure 3.2):

$$\beta = \tan^{-1}\left(\frac{-U}{V}\right) = \tan^{-1}\left(\frac{-9.4}{12.9}\right) = -36.08^\circ$$

Using equation 3.5:

$$|\gamma| = |\beta| + |i_{stall}| \implies |\gamma| = 36.08 + 14 = 50.08^\circ$$

Again using the velocity triangle to get the sign convention correct $\gamma = -50.1^\circ$

3.3 Turbine Power Control

Too high a power output from wind turbines is actually very undesirable. Too high a power output can over-stress the blades causing structural failure and the generator in the turbine has only a finite capacity to absorb power. Two methods are commonly used to control the power output.

1. Stall Control. The blade is designed mechanically for maximum lift. At higher wind speeds, the blades will stall, causing a reduction in lift and hence tangential force. The drag will be increased significantly, which contributes to the axial force, so there is a need to ensure the structure can endure the increased axial loading. The drawback of this technique is that predicting the onset of stall and the flow around a stalled aerofoil is very difficult.
2. Pitch Control. The blade is provided with an actuated mechanism to vary the blade angle at different wind conditions. When the wind speed is too high, the blade angle is altered to reduce incidence. This is shown in Figure 3.7 in Case A on the left the wind turbine blade angle γ is set so that the incidence i is large. In Case B on the right the blade angle has been set so that the incidence i has been reduced. The inset in the bottom left of Figure 3.7 shows the two blade shapes superimposed to highlight the differences between them. In low wind conditions pitching can be used to increase the incidence and hence reduce the power output. The drawback here is that a complex actuation mechanism has to be provided.

3.4 Further Reading

The method outlined is this Chapter is a simple method of calculating wind turbine performance. The simple wind turbine analysis assumes that the wind turbine does not alter the flow going through it when momentum changes are considered later it will be clear that this is only a first order approximation. A more accurate technique is the so called “Blade Element Momentum Method” which allows for the variation of flow over the blade and is described in much more detail in Manwell et al. (2002) along with much more detail on the general operation of wind turbines. Hansen and Butterfield (1993) provide an excellent description of some early development of the aerodynamics of wind turbine blades.

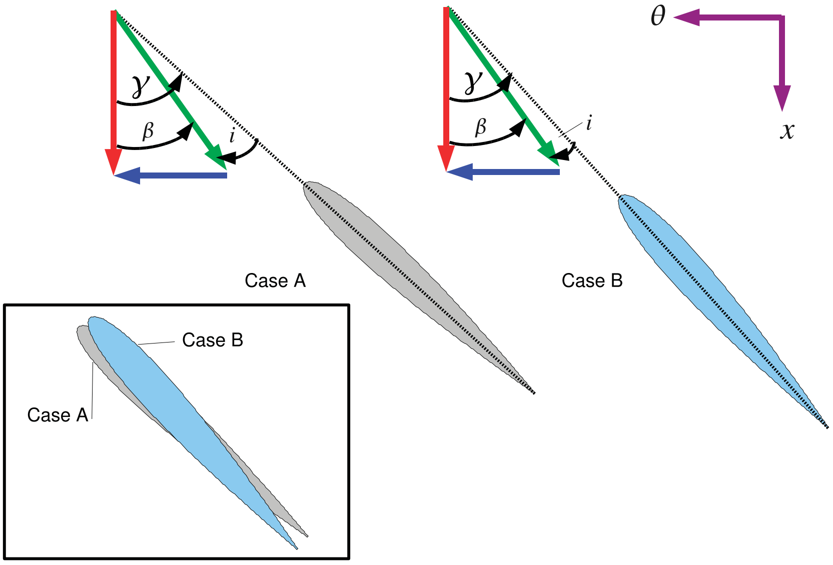

Figure 3.7: Schematic Showing Wind Turbine Pitch Control

3.5 Problems

1. A two-blade wind turbine is designed to operate at an atmospheric condition (the air density can be taken as $1.22\ kg/m^3$) with a wind speed of $22\ mph$. The turbine blades of $10\ m$ length are attached to a nacelle of a radius of $1\ m$. A preliminary blading design is considered by using a NACA 0012 profile with a constant chord length of $1.5\ m$. The rotational speed of the turbine is $30\ rev/min$. The blading design is taken at a condition corresponding to 80% of the maximum lift.

   Calculate the blade angle and the power output per unit blade-span at 20%, 50% and 80% spanwise sections for each blade. Note that the blade starts at radius of $1\ m$ so the tip radius is $11\ m$. Estimate the total power output of the wind turbine using the results from the three spanwise sections and an approximate integration.

   Answers: $-53.8°$; $-72.4°$;$-80.8°$; $1.201\ kW/m$; $3.75\ kW/m$; $7.92\ kW/m$; $86kW$

Chapter 4

Different Turbomachines and Their Operation

Now that the concept of an axial flow turbine has been introduced in some detail the geometries and mode of operation of a number of other types of turbomachine will be considered and the cascade view and velocity triangles applied to those machines.

There are three ways of classifying turbomachines, the first is by the type of fluid they work on and the second is based on the direction that the flow travels through the machine and the third is whether or not they deliver power to or extract power from the working fluid.

The classification by type of fluid splits machines into two categories. Those that work on com-

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pressible fluids (most often air or steam) such as aircraft engines, stationary gas turbines, steam power plants and high speed fans. The complication with these machines is that if the velocity of the fluid goes above the local speed of sound in the fluid shock waves may result. The second category is those that work in incompressible fluids such as water or oil such as pumps or hydraulic turbines. Shock waves are not an issue in liquids but the complication here is a phenomenon known as *cavitation* which occurs if the pressure of the fluid falls below a certain pressure. This is discussed in Chapter 10. Machines such as wind turbines and low speed fans fall outside this classification system as although the fluid is compressible in practise the changes in density are minimal so they can belong to either category.

The classification of machine by flow direction is more complex, essentially if the flow direction is roughly along the direction of the axis of the machine the device is called an “axial” machine but if the flow direction turns and has a component substantially in the radial direction the machine is know variously as a “radial” or “centrifugal” machine. The meaning of this classification will become clearer as the different machine types encountered in practise are described.

The final classification is that some turbomachines absorb power from the fluid (turbines) and some deliver energy to the fluid (compressors,fans or pumps).

4.1 Axial Flow Machines

A simple axial flow machine was used to introduce the idea of a turbomachine in Chapter 1 essentially an axial flow machine is one in which the fluid remains parallel to the axis of rotation as it passes through the machine. There are many examples of this type most aircraft engines use axial flow devices as do all large power station equipment and wind turbines are the most visually striking example.

The complication with axial flow machines compared to the simple one see in Figure 1.2 is that more than one stage may be present. This idea allows energy to be extracted in “small bites” from the machine allowing very large pressure ratios to be used in a single device - indeed it was this concept that is the key to the superiority of steam turbines compared to reciprocating engines. A single stage is sufficient when the fluid does not have a great deal of energy to be extracted for a turbine or when the requirement is for a modest pressure rise such as in most fans.

4.2 Radial and Centrifugal Flow Machines

To introduce the idea of a radial flow machine we will discuss a specific instance, that of a radial flow pump. A general arrangement drawing of the pump is shown in Figure 4.1, on the left the cross section of the pump is shown and on the right a plan view. Two components are shown in each view the outer casing also called the “volute” or “scroll casing” the aim of which is to ensure an even distribution of flow out of the machine, the area is therefore gradually increased up to the discharge pipe. The second component is the rotor also called the “impeller”.

The operation of the machine is such that flow approaches the device through the inlet pipe in the axial direction, the fluid is then turned through 90° to the radial direction where it enters the impeller. The rotor then increases the angular momentum of the fluid and it exits in the radial direction into the volute. The pump shown in Figure 4.1 has no stator following the rotor, the fluid slows and experiences a rise in static pressure simply by the action of the increased cross sectional area of the

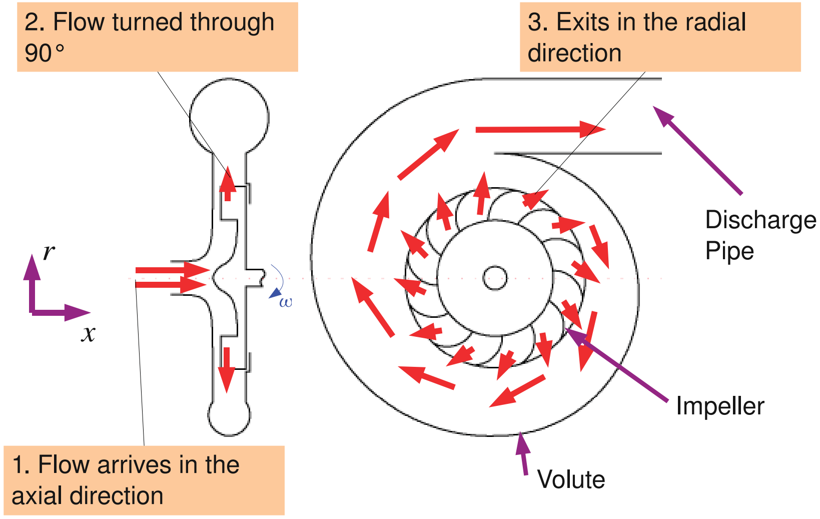
2. Flow turned through 90°
3. Exits in the radial direction
Discharge Pipe
r
x
ω
Impeller
1. Flow arrives in the axial direction
Volute

Figure 4.1: Radial Pump

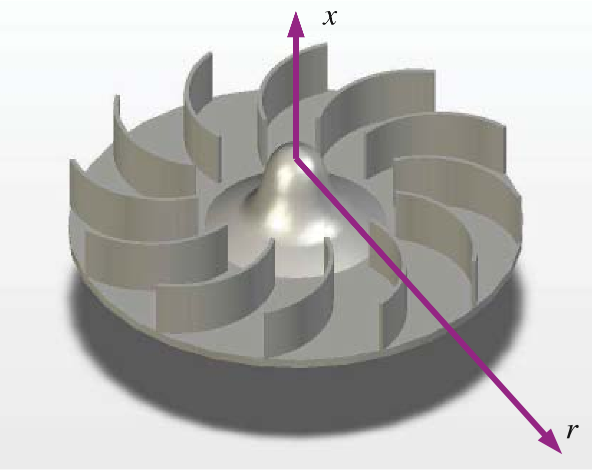

Figure 4.2: 3D View of the Radial Impeller

volute as the radius increases (An analysis of this can be found in Chapter 10) for large pressure rises it is necessary to put in a stator row to control this process.

A 3D view is shown in Figure 4.2 of the impeller only with a radial and the axial direction marked on, this hopefully puts Figure 4.1 in context. The blades of the impeller in Figure 4.2 are what is known as radial blades, that is no part of them operate in the axial-tangential plane. The impeller as a whole is therefore sometimes called a “radial impeller”. This is best seen in comparison to what is often called a “centrifugal impeller” shown in Figure 4.3 here the blades extend from the radial direction until they also have an axial component.

Although radial machines are most often found in pumping applications, radial turbines are

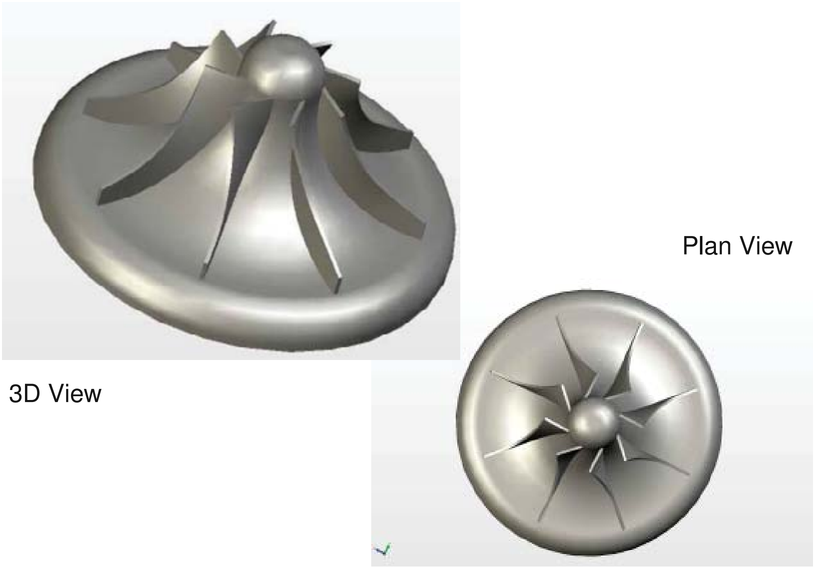

Figure 4.3: Centrifugal Impeller

widely used in small gas turbines such as the auxiliary power unit found on most aeroplanes and centrifugal designs and are used exclusively for turbochargers on internal combustion engines.

4.3 Radial Impellers

The velocity triangle can be applied to radial turbomachines in an identical manner for the axial flow machine used earlier. The difference is that the cascade view becomes slightly more abstract and the velocity triangles may be drawn in the radial tangential plane as well as the axial tangential plane.

First consider the radial impeller shown in Figure 4.1 and Figure 4.2. The cascade view for the axial flow turbine can be considered as an "unwrapping" of the turbine so that the rotational motion of the blades ω was transformed into a linear motion with magnitude ωr_m. We apply an identical process to the radial impeller in Figure 4.4 as we did for the axial machine in Figure 1.6 on page 18.

Figure 4.4 shows three views, the first is a 3D view of the radial impeller, the second shows a sketch of the turbine as viewed from upstream with the blades, axial inlet and blade tips highlighted. To form the cascade view we can approximate the real turbine with a tip radius $R = 0.15m$ and a spacing between the blades of s with a machine with an infinite radius and the same blade spacing (or pitch) of s. For the radial impeller the cascade view is formed by simply examining the sketch produced at the bottom of Figure 4.4 which shows the inlet and the exit from the blades.

There are two differences between the cascade view for a radial machine and the cascade view for the axial machines:

1. As we shall see later in Chapter 5 there are only two radii that we need to examine to determine the work input that the pump applies to the fluid: the radius at where the blades start and the radius at the blade tip. Unlike for axial machines the cascade view need only be drawn once no matter how large the radius of the machine.

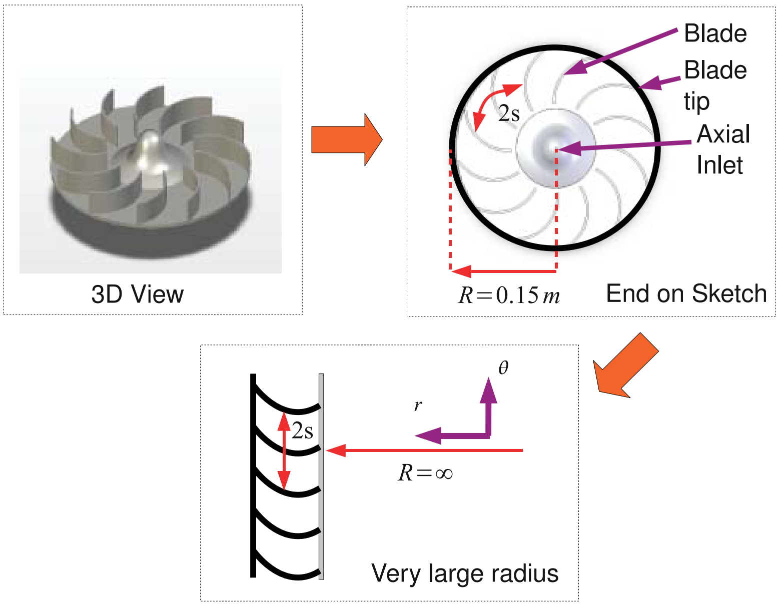
3D View
Blade
Blade tip
Axial Inlet
2s
$R = 0.15\,m$
End on Sketch
θ
r
2s
$R = \infty$
Very large radius

Figure 4.4: The Cascade View for a Radial Impeller

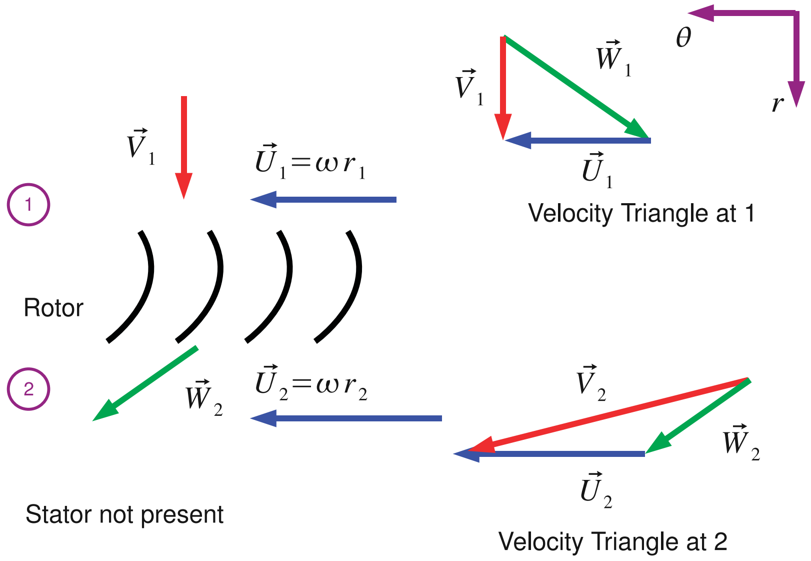
$\vec{V}_1$
$\vec{U}_1 = \omega r_1$
①
Rotor
②
$\vec{W}_2$
$\vec{U}_2 = \omega r_2$
Stator not present
$\vec{V}_1$
$\vec{W}_1$
θ
r
$\vec{U}_1$
Velocity Triangle at 1
$\vec{V}_2$
$\vec{W}_2$
$\vec{U}_2$
Velocity Triangle at 2

Figure 4.5: Velocity Triangles for a Radial Impeller

2. The cascade view for radial machines is formed in the radial tangential plane and not the axial tangential plane.

Having developed the cascade view the four step method for drawing velocity triangles (Section 2.3) is applied to the radial impeller at inlet and exit, this is shown in Figure 4.5 which shows the sketch of the cascade view and the two velocity triangles.

The cascade view is the same as Figure 4.4 simply turned through 90°. The inlet to the rotor is station 1 and the exit from the rotor is station 2 there is no stator in this case so no station 3 is drawn. A station 3 does exist however at the volute casing and Chapter 10 will show how to calculate the conditions using conservation of angular momentum - for now we stick to the velocity triangles. For radial machines the blade velocity $\vec{U}$ is *different* at station 1 and station 2. The blade velocity is given by $\vec{U} = \omega r$ and will get larger with increasing radius at the same revolutions per minute.

First consider the velocity triangles at station 1. The first step of the four step rule (Section 2.3) is to draw the flow that you know. At station 1 the flow that we know is the absolute inlet velocity at inlet to the impeller. There is no mechanism for the fluid to have been imparted with any tangential velocity (the inlet flow in the pipe is assumed to have no rotation in it) so the flow is purely radial. So a purely radial vector $\vec{V_1}$ be drawn. The second step of the four step rule is to draw the blade speed, the blades move in the tangential direction which is perpendicular to the radial direction so we can draw a vector $\vec{U_1}$. Step three is to close the triangle with the remaining vector $\vec{W}$. Finally check the correct orientation of the triangle by following the line drawn by $\vec{W}$ then $\vec{U}$ this should produce the same movement as following the vector $\vec{V}$. In Figure 4.5 the velocity triangle is correct so there is no need to adjust.

The velocity triangles at station 2 are drawn with exactly the same method. Step 1: the flow that is know this time is the relative velocity, in this book we assume that the flow always follows the blade geometry directly so once the blade has been sketched the relative velocity vector $\vec{W_2}$ at exit from the impeller may be drawn. Step 2: The blade speed is always purely tangential but note that the blade speed at 2 is greater than at station 1. Step 3: This time close the triangle with the absolute velocity vector $\vec{V_3}$. Finally in Step 4 check that the key rule $\vec{V} = \vec{U} + \vec{W}$ applies by following vectors.

An alternative approach to step three and four which is essentially a trial and error approach to getting the velocity triangle correct is to work out which way around the triangle should go using vector addition and get it right first time. The author's experience is that many students find velocity triangles very confusing at first and a slower but fail-safe method is more appropriate when first introduced to the concept.

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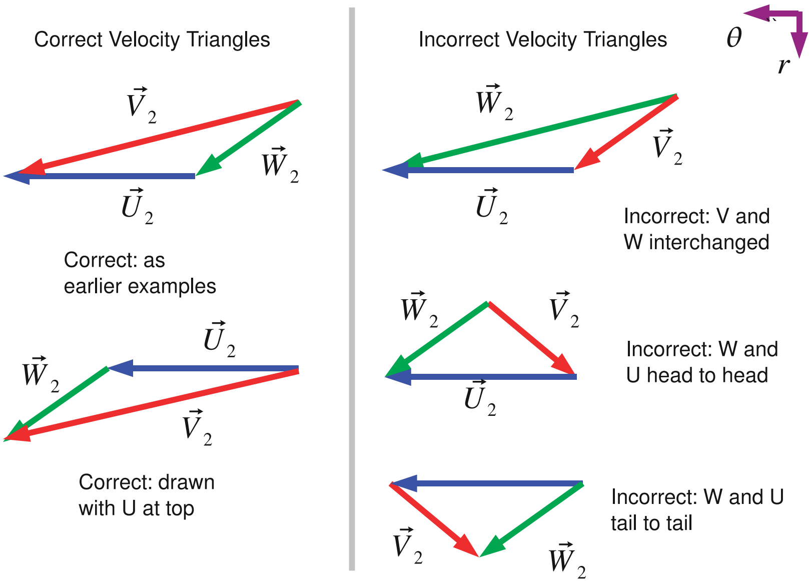
Correct Velocity Triangles
$\vec{V}_2$
$\vec{W}_2$
$\vec{U}_2$
Correct: as earlier examples
$\vec{U}_2$
$\vec{W}_2$
$\vec{V}_2$
Correct: drawn with U at top
Incorrect Velocity Triangles
θ
r
$\vec{W}_2$
$\vec{V}_2$
$\vec{U}_2$
Incorrect: V and W interchanged
$\vec{W}_2$
$\vec{V}_2$
$\vec{U}_2$
Incorrect: W and U head to head
Incorrect: W and U tail to tail
$\vec{V}_2$
$\vec{W}_2$

Figure 4.6: Common errors in Velocity Triangles

Figure 4.6 shows correct and incorrect velocity triangles for the radial impeller exit. On the left that are two correct triangles the difference being in whether the blade speed is above or below the other vectors. Note how the angles and lengths in both triangles are the same. The right hand side of Figure 4.6 shows a number of velocity triangles that are *incorrect*. The first has the relative and absolute velocity interchanged, the second two have the vectors for relative velocity and blade speed connected in an incorrect manner. The incorrect velocity triangles do not obey the key rule: $\vec{V} = \vec{U} + \vec{W}$ a tool which gives a systematic method for checking velocity triangles.

4.4 Centrifugal Impellers

We now apply the same techniques for deriving velocity triangles to the most complex case commonly encountered in turbomachinery - that of a turbomachine where the flow is turned from axial to radial *and* the blades operate in both the axial tangential and the radial tangential plane. The velocity triangle at inlet and exit are in different planes. The construction of the cascade view is illustrated in Figure 4.7 as before.

In the top left of Figure 4.7 the 3D view of the centrifugal impeller is shown. The front on “sketch” is show on the top right, hopefully the relationship between the two is clear as since a 3D sketch is hard to do a front view of the 3D model is used. To draw the cascade view two views in the radial-tangential plane are shown at the bottom of the figure.

The bottom left of Figure 4.7 shows a sketch of the blades when they are unwrapped to an infinite radius. The blade tip view is straightforward as it is the same as the blade tip view for the radial machine, the complication is that since the geometry is complex the section at the tip only is sketched.[1]

The blade hub view (bottom right of Figure 4.7) is actually very similar to the one found in Figure

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[1] Chapter 5 will show to determine the performance of a turbomachine we only need information at the inlet and exit of a set of blades.

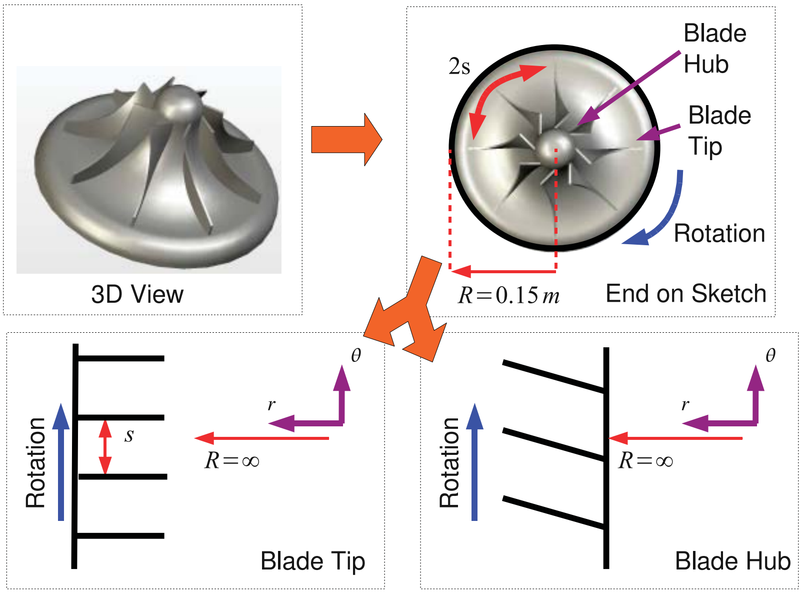

Figure 4.7: Constructing the Cascade View for a Centrifugal Impeller

1.6 except the sketch only covers the front part of the blades, the parts that are directly facing us in the top right view in Figure 4.7. It is apparent that since the blades extend outwards in the radial direction that there will be some variation in conditions in the radial direction at inlet to the blades. The blade speed ωr will increase with increasing radius. Normally centrifugal impellers are small enough to use a mid-height radius as representative of the flow at inlet but for detailed design work or for machines with very large blades a number of different radii should be examined at inlet. The cascade view is formed (as for an axial flow turbine) by looking down from the casing to the hub so you get a plan view of the blades at the inlet to the impeller.

The two different cascade views are produced in Figure 4.8 along the corresponding velocity triangles at inlet and exit. The triangles are built up using the four step rule, the process for which is the same as for radial machines.

For station 1 the cascade view is sketched, it is important to remember that the cascade view is only for the entry to the machine, the complex geometry that extends beyond the start of the rotor row blading is ignored. The cascade view at inlet is sketched in the axial-tangential plane. At station 1 the known flow is the absolute velocity which with the absence of any device to increase angular momentum will have zero tangential velocity. The blade speed is given by ωr_1 where r_1 is mean radius at station 1 as discussed earlier. With the blade speed and absolute the relative velocity can be used to close the triangle and the velocity triangle is complete.

For station 2 the view is the axial-radial plane and the known velocity is the relative velocity leaving the blades. The blades in the centrifugal impeller shown are radial at exit or to put it another way the blade direction at exit is parallel to the radial axis. Since the blades are radial the relative velocity at exit is also radial. The blade speed is given by ωr_2 which since $r_2 > r_1$ will be greater than the blade speed at inlet. The relative velocity and blade speed allow to complete the velocity triangle.

Figure 4.8 represents the most complex velocity triangle and cascade view encountered in practise so once the reader has mastered this technique you are well equipped to analyse almost any turbomachine.

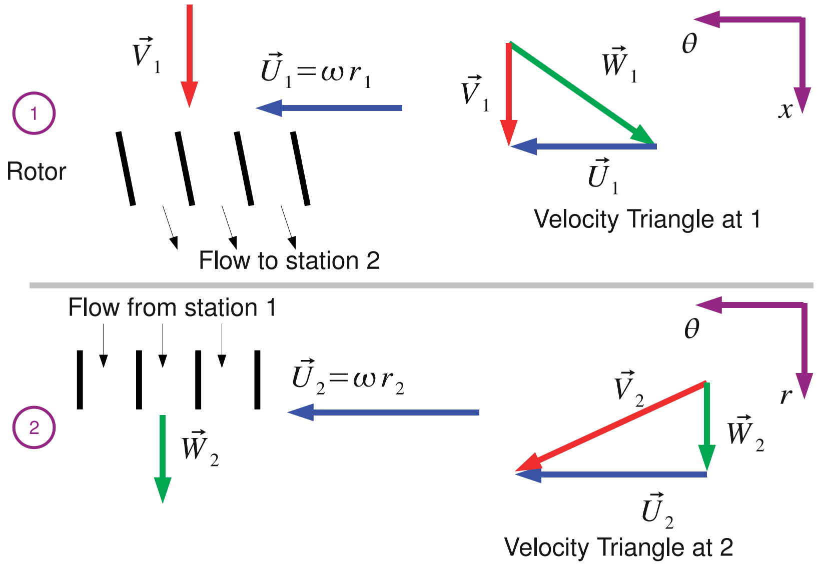

Figure 4.8: Velocity Triangles for a Centrifugal Impeller

Note that the analysis of radial flow machines is not restricted to pumps, the technique is equally applicable to hydraulic turbines, turbochargers and so on.

4.5 Hydraulic Turbines

Hydraulic turbines are devices for extracting energy from a reservoir of fluid usually obtained from rainfall above sea level. Water is collected in a reservoir at a height then is carried by a series of pipes to a turbine which discharges into a river (Figure 4.9). Hydraulic turbines are universally used to generate electricity and there are broadly three types of hydraulic machine in use: The Pelton wheel, Francis turbine and the Kaplan turbine, each of which were named after their inventors are are in common use around the world today.

Using the relationship $\Delta p = \Delta h \rho g$ it is possible to express any pressure in terms of height of a particular fluid. This is an approach commonly used in hydraulic turbines as pressures directly relate to the height of the reservoir or the height above river level that the turbine is situated at.

Hydraulic turbines are usually classified in terms of *total head*. For any position in a fluid system the total head is given by the following equation:

$$H = \frac{p}{\rho g} + \frac{V^2}{2g} + z \qquad (4.1)$$

If we examine the total head at the dam of a reservoir (Station 0 in Figure 4.9) where the height datum is the river level at exit from the turbine. The surface area is very large so $V \approx 0$ and since the pressure is at atmospheric pressure $p = 0$. The total head is therefore given entirely by the height term z. At entry to the turbine however the height term $z \approx 0$ but the pressure and velocity terms will be much larger - essentially gravitational potential energy has been traded for pressure and velocity.

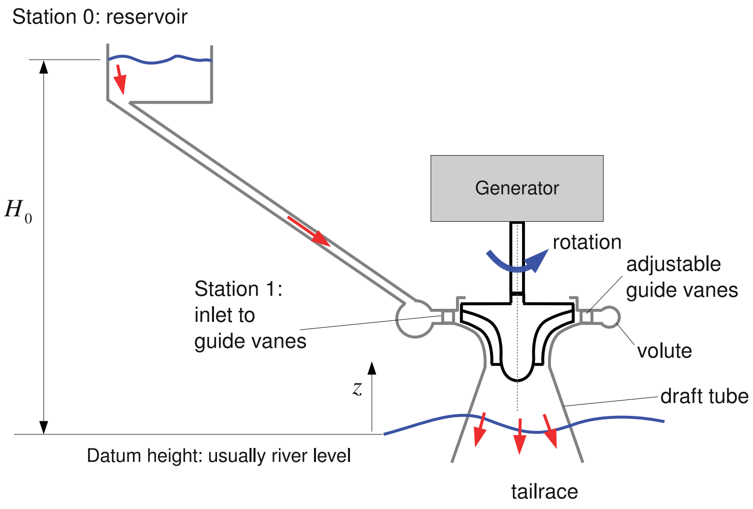
Station 0: reservoir
H_0
Generator
rotation
adjustable guide vanes
Station 1: inlet to guide vanes
volute
z
draft tube
Datum height: usually river level
tailrace

Figure 4.9: Schematic of Hydro-Electric Scheme

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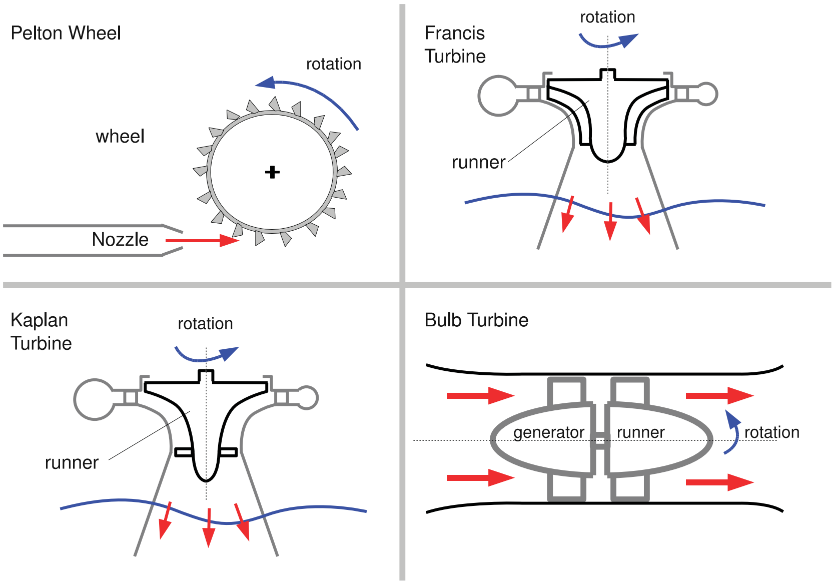

Figure 4.10: The Four Major Types of Hydraulic Turbine

Assuming that there are no losses the maximum velocity that would result from a nozzle placed next to the turbine open to the atmosphere can easily be found:

$$\frac{p_0}{\rho g} + \frac{V_0^2}{2g} + z_0 = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1$$

But $z_1 \approx 0$ and $p_0 = p_1 = 0$ and $v_0 \approx 0$ therefore:

$$z_0 = \frac{V_1^2}{2g} \implies V_1 = \sqrt{2gz_0}$$

The four major types of turbines are shown schematically in Figure 4.10.

The Pelton wheel is the simplest form of water turbine and is used for high head installations. The Pelton wheel consists of one or more nozzles which produce a high velocity jet of water these then impact a series of buckets which divide the flow in two and the flow exits to the side of the machine. The original patent from Pelton in 1880 is shown in Figure 4.11 which includes details of his bucket design. The analysis model of the blade is shown on the right hand side of Figure 4.11 essentially the blade experiences a relative velocity W_1 onto the blades which is then turned through some angle and leaves with velocity kW_1 where k is an empirical coefficient for the reduction in velocity due to friction with the bucket etc. Analysis of the Pelton wheel is most convenient when the angle Θ is used in the analysis. The flow is turned through $180° - \Theta$ and so the Pelton wheel is a high turning device as typical values of Θ are ten to twenty degrees.

The Francis turbine is used for moderate head installations and in many ways can be though of as a centrifugal impeller working in reverse. A cross-section of the device is in Figure 4.10 flow enters through a volute or scroll casing which is designed to evenly distribute the flow around the periphery of the inlet guide vanes. The inlet guide vanes increase the angular momentum of the fluid which is reduced again in the turbine rotor that turns the flow from the radial to the axial direction. The flow then exits into a draft tube and from there to a river.

Figure 4.12 shows a 3D CAD drawing of a Francis turbine, the real machine is quite complex so an external view and a view with the top half of the turbine removed are shown. The Francis turbine shown has two sets of guide vanes the first at a high radius are fixed in place, the second allow control of the machine by being pivoted on an axis. Usually these guide vanes can turn to 90° so can actually stop all flow through the machine to allow it to started and stopped. The control mechanism for these vanes is clearly seen in the external view. The relationship between the three dimensional object and the sketch in Figure 4.10 should be clear.

The Kaplan turbine is used for low head applications, the operation is very similar to that of a Francis turbine but the rotor design is different. The blades do not extend to the radial direction, so the flow is turned before it enters the rotor blade row. This can be seen in Figure 4.13. The Kaplan turbine impeller is a lot like that of a propeller. The bulb turbine is very similar to an axial flow turbine, except that the medium is water rather than a gas. Again both turbine types are shown in Figure 4.10.

4.6 Common Design Choices

One common design choice is to have *constant axial velocity*: $V_{1x} = V_{2x} = V_{3x}$

Hence the blade height for turbine must increase as density falls, as seen earlier. There is no fundamental physical law that produces this constraint it is a design *choice*, although it is often used.

For multi-stage machines, successive stages are often designed to have identical flow angles and velocities at corresponding positions in each stage:

$$V_1 = V_3 \implies \alpha_1 = \alpha_3 \text{ and } V_{1\theta} = V_{3\theta}$$

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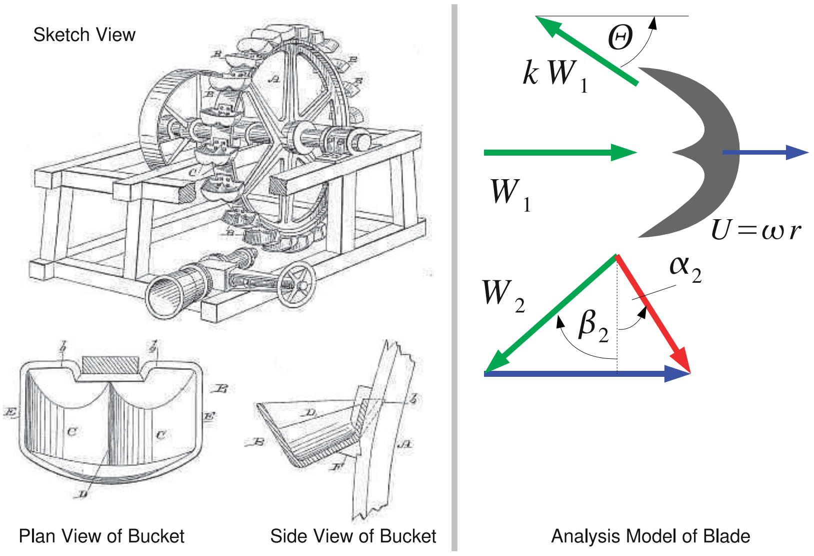
Sketch View

Plan View of Bucket

Side View of Bucket

Analysis Model of Blade

Θ
$k\,W_1$
W_1
$U=\omega r$
α_2
W_2
β_2

Figure 4.11: Pelton's Patent Application and Analysis Model

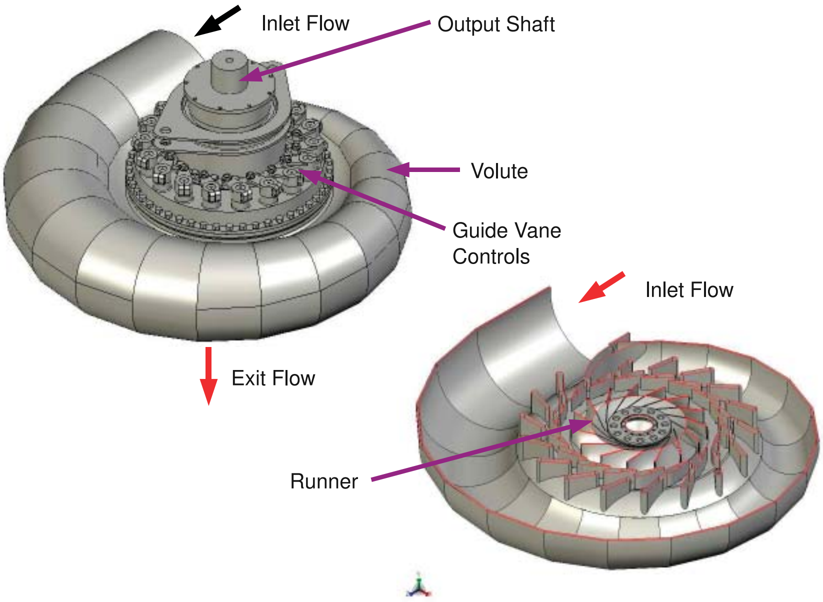
Inlet Flow
Output Shaft
Volute
Guide Vane Controls
Inlet Flow
Exit Flow
Runner

Figure 4.12: Three Dimensional Views of a Francis Turbine

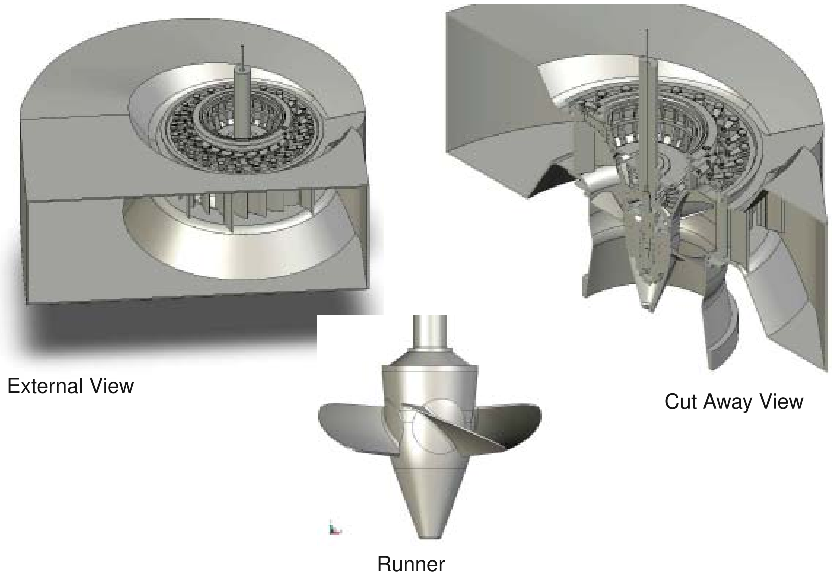

Figure 4.13: Three Dimensional Views of a Kaplan Turbine

This is know as a *Repeating Stage* where the velocity at inlet is equal to the velocity at exit.

Often we design for *axial leaving velocity* from a turbomachine. Since for a given V_x the exit velocity V_3 is given by $V_3 = \sqrt{V_x^2 + V_{\theta 3}^2}$ the minimum exit kinetic energy at exit occurs when $V_{\theta 3} = 0$ or the flow is entirely in the axial direction. Clearly this condition occurs at $\alpha_3 = 0$ so an axial leaving condition specifies the shape of the exit velocity triangle for the stator or rotor row.

These three commonly used design conditions are design choices, although there are many successful turbomachines in use that do use these design choices there are some that do not.

4.7 The Turbomachine and System

The turbomachine always operates as part of a system. Pumps are required to deliver fluid at a higher pressure for water delivery for example, steam turbines receive steam from a boiler and deliver it to a condenser or some industrial process. The torque from a turbine is either used to generate electricity or provide mechanical power for gearboxes. The mechanical work for a compressor or a fan comes from either electrical power (or in the case of a gas turbine engine is taken directly from the turbine). So both the fluid movement and the work changes from the machine fit into a system.

The apparent exception to this rule are devices such as wind turbines which although they supply electricity to some sort of load have varying and unpredictable inlet conditions. Even here a great deal of work is done to ensure that turbines are sited appropriately so that they extract the maximum amount of energy at a given location and have a minimal visual impact.

The inputs and outputs from the turbomachine are not arbitrary and are usually fixed by external parameters:

- For steam turbines the inlet conditions are fixed by the boiler conditions which are dependant on the type of fuel used in the cycle and the overall thermodynamic cycle that is chosen for the site. In a steam cycle the exit conditions are fixed by the temperature at which the condenser cooling fluid can operate at.

- For aircraft engines the inlet conditions are determined by the operating envelope of the aeroplane and the exit conditions largely determined by the amount of thrust required from the engine.
- For pumps the exit conditions are set by the flow rate required and pressure loss introduced by the piping system to which the pump is connected to. In the case of moving the fluid from one location to another the pump also has to supply a pressure equivalent to the change in height between the two locations as well as any losses in the system.
- For wind turbines the inlet conditions are set by the location of the wind turbine and the variability of the weather. Of all turbomachines wind turbines have the most variable inlet conditions and ensuring that the design performs over a wind range of inlet conditions is key to producing a successful turbine.
- For hydro-electric power systems the available pressure or head is determined by the height difference between the reservoir and the river into which the fluid is discharged. (See Figure 4.9) The flow-rate is determined by the rainfall for the region in which the hydro system is located and the capacity of the reservoir. A common strategy is to use the reservoir as an energy storage system and run the turbines when electrical demand is high, turn them off during periods of low demand and allow the reservoir level to rise.

How the turbomachine fits into the system is generally beyond the scope of this book but it is worth remembering the impact that the system will have on the design of the turbomachine.

4.8 Problems

1. Sketch the cascade and meridional views for a centrifugal and a radial impeller.
2. For the Francis turbine in Figure 4.12 sketch the cascade for inlet and exit. Sketch also the meridional view.
3. For the Kaplan turbine in Figure 4.13 sketch the cascade for inlet and exit. Sketch also the meridional view.

Chapter 5

Application of The Equations of Fluid Motion

Engineering study of fluid dynamics (and turbomachinery) is built on three sets of equations the conservation of mass (continuity), conservation of momentum and the conservation of energy.

5.1 Conservation of Mass

Conservation of mass can be simply expressed as "what goes in must come out", in steady state operation the mass flow of the fluid entering the machine must be matched by mass flow exiting the

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In equation form this is expressed as:

$$\dot{m} = \rho A V \tag{5.1}$$

Where $\dot{m}$ is the mass flow, ρ is the fluid density and A is the cross sectional area and V is the velocity. For the equation to be valid the velocities and cross sectional areas *must be perpendicular to each other*. This will become clearer with the application of the equation to various machines.

The simplest case to examine is that of an axial flow turbine. Consider the meridional view of the machine shown on the right hand side of Figure 2.4. We denote (as before) the conditions at each station by a subscript so ρ_2 is the density at station 2 for example. Conservation of mass tells us that:

$$\rho_1 A_1 V_{1x} = \rho_2 A_2 V_{2x} = \rho_3 A_3 V_{3x} \tag{5.2}$$

Note that the axial component of velocity is used, as this is perpendicular to the cross sectional area under consideration. The cross sectional area of an axial flow turbine is given by the area of an annulus between the hub radius r_h and the tip radius r_t:

$$A = \pi(r_t^2 - r_h^2) \tag{5.3}$$

In turbomachinery applications it is more useful to express the area in terms of the mean radius r_m and the blade height b.

From Figure 2.4 we can see that:

$$r_m = \frac{r_t + r_h}{2} \implies 2r_m = r_t + r_h \tag{5.4}$$

and

$$b = r_t - r_h \tag{5.5}$$

We can rewrite the area (Equation 5.3) and substitute using Equations 5.4 and 5.5:

$$A = \pi(r_t^2 - r_h^2) = \pi(r_t + r_h)(r_t - r_h) = \pi(2r_m)(b) = 2\pi r_m b$$

There are therefore two alternative expressions for the cross sectional area in an axial flow turbine:

$$A = \pi(r_t^2 - r_h^2) = 2\pi r_m b \tag{5.6}$$

Conservation of mass provides us with a powerful tool to explain the geometry of many turbomachines.

Example An industrial turbine operates at an 8.8:1 pressure ratio and a mass flow of $77\,kg/s$ using air as the working fluid. The exhaust temperature is at $43^\circ\,C$ and the inlet temperature to the machine is around $1000^\circ\,C$. The mean blade radius is $0.4\,m$. The machine is to be designed for a constant axial velocity of $200\,m/s$. Estimate the blade heights at entry and exit of the turbine.

Solution This problem requires us to apply the continuity equation at inlet and exit from the turbine, to do this we need to know the density of the fluid at inlet and exit. This is obtained from basic fluid dynamics:

$$\frac{p}{\rho} = RT \tag{5.7}$$

where p is pressure, R is the gas constant and T is the temperature. For air $R = 287\ J/kgK$ is a good approximation. The temperature has to be expressed in Kelvin.

At entry to the turbine:

$$p_1 = 8.8\ bar = 8.8 \times 10^5\ Pa \text{ and } T_1 = 1000 + 273 = 1273\ K$$

At exit from the turbine:

$$p_2 = 1.0\ bar = 1.0 \times 10^5\ Pa \text{ and } T_2 = 473 + 273 = 730\ K$$

The corresponding densities are therefore:

$$\rho_1 = \frac{p_1}{RT_1} = \frac{8.8 \times 10^5}{287 \times 1273} = 2.41\ kg/m^3$$

$$\rho_2 = \frac{p_2}{RT_2} = \frac{1.0 \times 10^5}{287 \times 710} = 0.49\ kg/m^3$$

Finally we apply the continuity equation to obtain the required blade heights:

$$\dot{m} = \rho A V_x = \rho 2\pi r_m b V_x \implies b = \frac{\dot{m}}{\rho 2\pi r_m V_x}$$

Since mass flow conservation applies to the turbine (what goes in must come out!). This can then be applied to the entry and exit of the turbine.

$$b_1 = \frac{77}{2.41 \times 2\pi \times 0.4 \times 200} = 0.06\ m$$

$$b_2 = \frac{77}{0.49 \times 2\pi \times 0.4 \times 200} = 0.31\ m$$

This is shown approximately to scale in Figure 5.1. The turbine is split into three stages each with a stator and rotor. The design is for a constant mean radius although it is equally possible to design for a constant inner or outer diameter. The key point is that Equation 5.1 can explain why the blade height changes through a typical gas turbine. This is based on the assumption that the axial velocity remains constant, a common design choice that is discussed later.

5.1.1 Application to Radial Machines

The application to radial machines is straightforward but requires a good understanding of the geometry of these machines. Figure 5.2 shows the meridional view of a radial and a centrifugal impeller. For the radial turbine (shown on the left) the application of continuity is straightforward. The flow

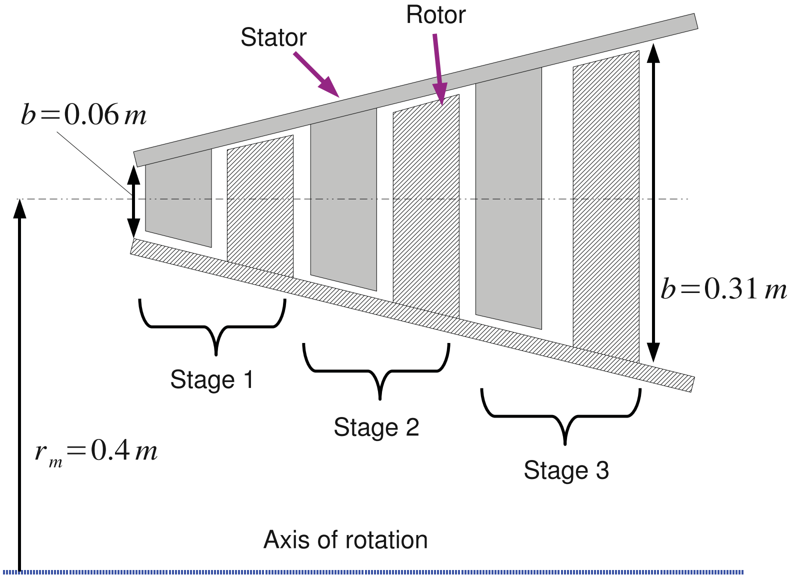

Figure 5.1: Meridional View of a Gas Turbine

perpendicular to the cross sectional area of interest is the radial velocity and the cross sectional area is given by the area of a cylinder face with a depth given by the blade height b and a circumference given by $2\pi r$, that is $A = 2\pi rb$ so the continuity equation becomes:

$$\rho_1 A_1 V_{1r} = \rho_2 A_2 V_{2r} \implies \rho_1 2\pi r_1 b_1 V_{1r} = \rho_2 2\pi r_2 b_2 V_{2r}$$

So for machines operating at constant density and constant radial velocity then $b_2 < b_1$. The blade height falls as the radius increases.

The application to centrifugal machines (right hand side of Figure 5.2) requires slightly more care. At exit the continuity equation is the same but at inlet different velocities and areas are needed. At inlet the velocity is the axial velocity and the perpendicular area is the annulus between the hub and the tip of the blades. So $A = 2\pi r_m b$ where r_m is the mean radius at inlet to the machine. Therefore the continuity equation becomes:

$$\dot{m}_1 = \rho_1 A_1 V_{1x} = \rho_1 2\pi r_{1m} b V_{1x}$$

and for station 2:

$$\dot{m}_2 = \rho_2 A_2 V_{1r} = \rho_2 2\pi r_2 b V_{2r}$$

The most useful application of these equations is that given a mass flow and a machine geometry the velocities inside the machine can be calculated.

5.2 Conservation of Momentum

Newton's second law of motion can be applied to rotational as well as linear systems in words: "torque is equal to the rate of change of angular momentum". In equation form this is given by:

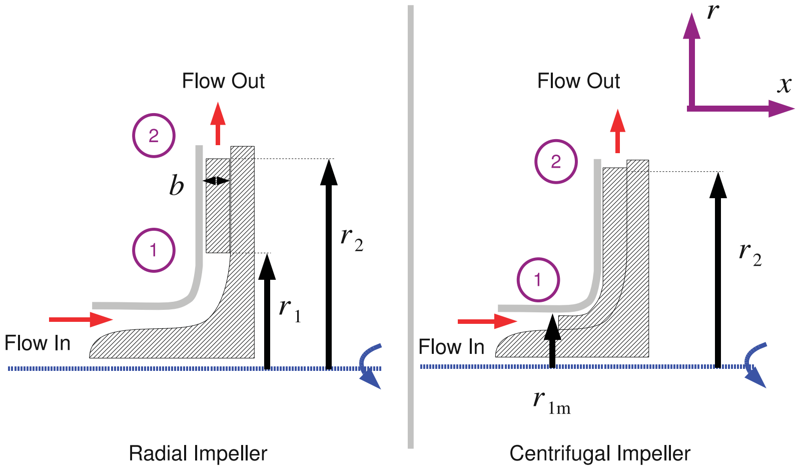

Figure 5.2: Meridional Views of Radial and Centrifugal Machines

$$T = \frac{dL}{dt} \tag{5.8}$$

Where T is the torque, L is the angular momentum and t is time. In linear motion force is equal to the rate of change of momentum in the system which is the well known form of Newton's second law.

We now apply Equation 5.8 to the flow through a turbomachinery passage shown in Figure 5.3 with an entry station 1 and an exit station 2. The passage varies in radius from station 1 to station 2 and can be any shape (though in the figure the radius change is linear), the passage is bounded by a hub and a casing so mass flow is conserved through the turbomachinery passage.

The velocity through this passage may differ between entry and exit but at either of those stations is uniform. The left hand side of Figure 5.3 shows a cross sectional view of the turbomachinery passage at entry and at the top a representative velocity vector decomposed into radial and tangential components is shown. This vector would have the same value of radial and tangential components no matter where on the circumference of the passage at entry we chose to measure it. This is illustrated by drawing the same vector at 3 o'clock, 6 o'clock and 9 o'clock on the cross section at entry to the annulus. A similar exercise can be carried out at exit from the turbomachinery passage.

Angular momentum is given by the moment of momentum or:

$$L = mV_\theta r \tag{5.9}$$

Where m represents the mass. So at entry to the turbomachinery passage the angular momentum is $m_1V_{1\theta}r_1$ and at exit from the passage is $m_2V_{2\theta}r_2$. So the change in angular momentum between entry and exit is:

$$m_2V_{2\theta}r_2 - m_1V_{1\theta}r_1$$

and the time rate of change is given by:

$$\frac{m_2 V_{2\theta} r_2 - m_1 V_{1\theta} r_1}{t} = \dot{m}_2 V_{2\theta} r_2 - \dot{m}_1 V_{1\theta} r_1 = \dot{m}(V_{2\theta} r_2 - V_{1\theta} r_1)$$

Since “what goes in must come out” $\dot{m}_1 = \dot{m}_2$. The time rate of angular momentum is equal to the torque we have:

$$T = \dot{m}(V_{2\theta} r_2 - V_{1\theta} r_1) \tag{5.10}$$

Recall from basic mechanics that power is torque times rotational speed then:

$$P = T\omega = \dot{m}\omega(V_{2\theta} r_2 - V_{1\theta} r_1) \tag{5.11}$$

Where ω is the rotational speed of the device. It is very useful in turbomachinery to work in terms of quantities per unit of mass going through the turbine, so we divide Equation 5.11 by the mass to give one of the most important equations in the book:

$$w = \omega(V_{2\theta} r_2 - V_{1\theta} r_1) \tag{5.12}$$

This is known as the *Euler Turbomachinery Equation*. The Euler equation applies to all types of turbomachinery, from wind turbines to pumps to gas turbines, whether they be axial flow, radial flow or a mixed flow device. It also applies to either energy extraction devices (turbines) or energy delivery devices (compressors, fans or pumps).

There are a number of important points that this equation brings to light:

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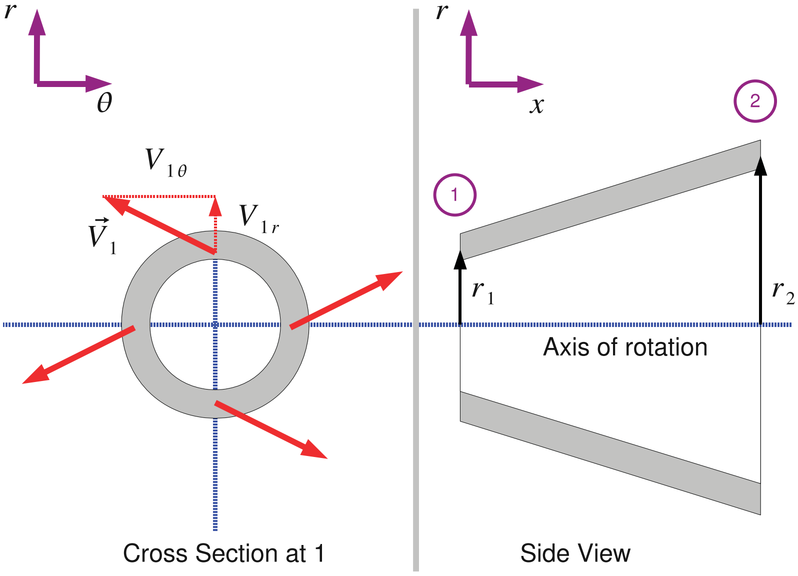

Figure 5.3: A Generic Turbomachinery Flow Passage

- For a turbine the Euler equation will give a negative value and for a compressor it will give a positive value. The energy flows are in different directions so it is reasonable that they be of different signs! It is common when dealing with turbines to multiply the right hand side of Equation 5.12 by -1 to avoid continually have to write negative signs in front of every calculation. Some authors (e.g. Dixon (2005)) refer to different forms of the Euler equation to deal with this circumstance but in this book we will stick with Equation 5.12.
- The power input or output is given entirely by the change in angular momentum. This is determined by the turning done by the fluid in the turbomachinery passage which is controlled by the blade angles of the turbomachine. In short: *work input or output is determined by the flow turning* - this is one of the fundamental concepts of turbomachinery and is vital to understanding the operation of all these important devices.
- Finally we note that since angular momentum (Equation 5.9) is the product of mass, radius and tangential velocity that changes in radius can also influence the power output or input. So radial flow machines tend to deliver or absorb much more power per stage than a corresponding axial flow device.

Example A turbine stage with a rotational speed of 3000 rpm is to be designed with an absolute inlet angle of $60°$ and an absolute exit angle of $-60°$ at a mean radius of $0.4\,m$. The machine is to be designed for a constant axial velocity of $450\,m/s$. Estimate the specific work from this stage.

Solution This is a straightforward application of the Euler work equation (Equation 5.12), but since the mean radius is constant $r_1 = r_2$ and we can simplify somewhat.

$$w = \omega(V_{2\theta}r_2 - V_{1\theta}r_1) = \omega r(V_{2\theta} - V_{1\theta}) = U(V_{2\theta} - V_{1\theta})$$

Here the blade speed is obtained easily:

$$U = 3000 \times \frac{2\pi}{60} \times 0.4 = 125.6\,m/s$$

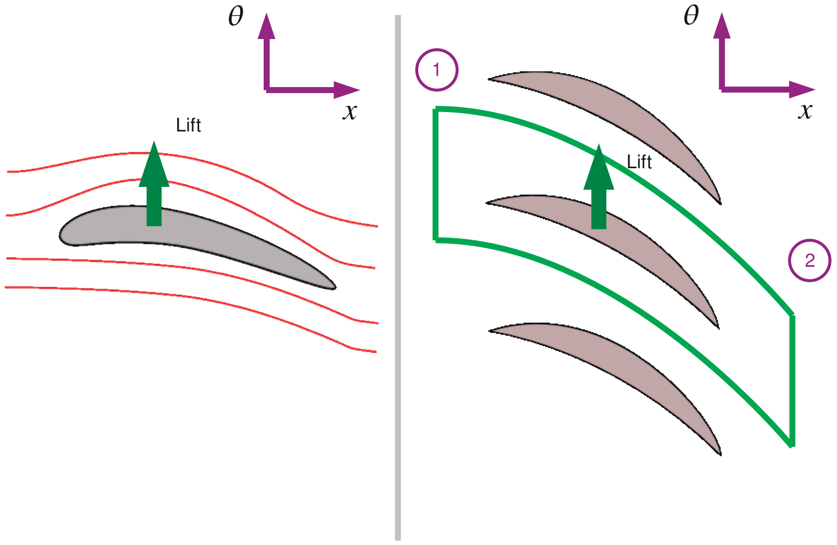

Figure 5.4: Isolated Aerofoil compared to a Cascade

The tangential velocities are obtained from the velocity triangle. This example is straight forward the absolute inlet and exit angles are given. From Chapter 2:

$$V_\theta = V_x \tan\alpha$$

So at inlet to the stage:

$$V_{1\theta} = 450 \tan 60 = 779 \, m/s$$

and at exit from the stage:

$$V_{2\theta} = 450 \tan -60 = -779 \, m/s$$

The specific work from this stage is given by:

$$w = 125.6 \times (-779 - 779) = -195.8 \, kJ/kg$$

5.2.1 The Difference Between a Single Aerofoil and a Cascade of Blades

This is a good point to explain the differences between a single aerofoil and the operation of a linear cascade. The operation of a single aerofoil is much closer to most common experience - nearly everyone has been on an aeroplane for example. In this book we do not discuss the detailed mechanisms of how an aerofoil works there are other excellent explanations of this phenomenon such as Babinsky (2003). Suffice to say that when an isolated aerofoil (Left hand side of Figure 5.4) meets an oncoming stream of fluid a lift force is produced. A small drag force is also produced. The aerofoil exerts and equal and opposite force onto the stream of fluid but since the mass of the incoming stream is effectively infinite the net deflection of the flow is zero.

In the case of the cascade shown on the right hand side of Figure 5.4 the aerofoil operates on a finite mass of air as the upper and lower blades of the cascade limit the area that a single blade can operate on. With a finite mass the reaction to the lift force produces a net deflection on the flow.

This actually provides an alternative derivation of the Euler Turbomachinery Equation. Consider the control volume drawn around the blade on the right hand side of Figure 5.4. The force exerted on the fluid is given by the rate change of momentum of the fluid - which is Newton's second law of motion. The momentum at stage 1 is $m_1\vec{V_1}$ and the momentum at stage 2 is $m_2\vec{V_2}$, since we are normally interested in the force in the tangential direction and the velocities are normally steady the rate of change of momentum and force in the tangential direction is given by:

$$F_\theta = \frac{m_2V_{2\theta} - m_1V_{1\theta}}{t} = \dot{m}(V_{2\theta} - V_{1\theta})$$

Torque is then given by the tangential force times the radius and power is the torque times the rotational speed:

$$T = F_\theta r_m \implies P = F_\theta r_m \omega = r_m\omega\dot{m}(V_{2\theta} - V_{1\theta})$$

Again we divide through by the mass flow to give the power per unit of mass flowing through the turbomachine:

$$w = \frac{P}{\dot{m}} = r_m\omega(V_{2\theta} - V_{1\theta}) \tag{5.13}$$

Equation 5.13 is a less general form of the equation obtained earlier using angular momentum (Equation 5.12) as it relies on the assumption of constant radius between entry and exit of the control volume - something that is not true of all turbomachine geometries.

5.3 Conservation of Energy and Rothalpy

The conservation of energy can be applied to a turbomachinery blade row as well, this leads to a new concept called rothalpy. For simple design a very large amount of information can be obtained without recourse to Rothalpy but the concept simplifies more advanced work.

The steady flow energy equation is applied to the flow between station 1 and station 2 of a turbine or a compressor. The blade may or may not be in a rotor row. The steady flow energy equation applied to the turbine shown in Figure 2.5 is given by:

$$\dot{q} + \dot{w} = \dot{m}\left[(h_2 - h_1) + \frac{1}{2}(V_2^2 - V_1^2) + g(z_2 - z_1)\right] \tag{5.14}$$

There are two simplifications that can be immediately applied to this equation. Firstly any height changes through the machine will be negligible compared to the changes in velocity and enthalpy so the $z_2 - z_1$ term will be zero.

Secondly the heat transfer to and from the fluid in the machine will also be zero, $\dot{q} = 0$. This second assumption requires some explanation as we also apply this to turbomachines where the temperature of the fluid is around 1500 K. The reasoning is that the velocity of the fluid is high enough that the amount of time a given particle of fluid spends inside the machine is very small so there is no opportunity for heat transfer for take place. In reality of course the outside of a high temperature turbomachine will get very hot but the heat transfer through the machine casing will be very small compared to the work transfers taking place inside the device.

Equation 5.14 therefore becomes:

$$\dot{w} = \dot{m}\left[(h_2 - h_1) + \frac{1}{2}(V_2^2 - V_1^2)\right] = \dot{m}\left[\left(h_2 + \frac{V_2^2}{2}\right) - \left(h_1 + \frac{V_1^2}{2}\right)\right]$$

If a moving fluid is brought to rest in an isentropic (or fully reversible) manner different values of temperature and pressure will be obtained than if measurements were taken with the fluid still moving. The values of temperature and pressure thus obtained are known as the *stagnation* conditions and those obtained whilst the fluid is moving are known as the *static* conditions. The same concept of static and stagnation conditions can be applied to the quantity enthalpy:

$$h_0 = h + \frac{V^2}{2} \tag{5.15}$$

where h_o is known as the stagnation enthalpy and h is called the static enthalpy.

Equation 5.14 can be further simplified to:

$$\dot{w} = \dot{m}\left[h_{02} - h_{01}\right] \tag{5.16}$$

Equation 5.16 is important as it relates the work output or input from the turbomachine to the change in fluid properties between the inlet and the exit of the machine. The method for determining h_0 varies depending on the type of fluid.

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5.3.1 Rothalpy

Note that $\dot{w}/\dot{m}$ is the specific work w so the equation is further simplified to:

$$w = h_{02} - h_{01}$$

But the Euler Turbomachinery Equation (Equation 5.12) also contains an expression for w, so Equation 5.14 becomes:

$$w = U_2 V_{2\theta} - U_1 V_{1\theta} = h_{02} - h_{01} \implies h_{01} - U_1 V_{1\theta} = h_{02} - U_2 V_{2\theta}$$

Define a new quantity called *rothalpy* given by:

$$I = h_0 - U V_\theta \tag{5.17}$$

From the re-arrangement of the SFEE energy equation, rothalpy is constant from station 1 to station 2, $I_1 = I_2$, which leads to the general principle that the conservation of energy applied to turbomachines can be expressed as “over a blade row rothalpy is constant”.

Rothalpy is conserved over a blade row (stator *or* rotor) and not a stage (stator *plus* rotor). The value of rothalpy at the stator/rotor interface will clearly change as the stagnation enthalpy h_0 will be the same but the blade speed U will be different depending on whether or not one is considering the rotor or the stator.

5.3.2 Rothalpy in Stators and Rotors

For a stator Rothalpy conservation is straightforward, since $U = 0$, rothalpy conservation from stator entry at station 1 to stator exit at station 2 yields:

$$I_1 = I_2 \implies h_{01} = h_{02}$$

Or in words - in a stator the stagnation enthalpy is conserved. For a rotor the situation is more complex. Consider a generic velocity triangle in Figure 5.5 from which we will derive some general relationships between relative and absolute velocity. Since the relationships are general no subscripts are used to indicated a particular station, the relationships could apply to any station. Some of these were discussed in Chapter 2 when the concept of velocity triangles was introduced but are worth repeating here.

Two relationships are obvious from the triangle:

$$V_x = W_x \text{ and } V_\theta = U + W_\theta$$

Pythagoras’s rule can be applied to the absolute velocity, $\vec{V}$:

$$V^2 = V_x^2 + V_\theta^2$$

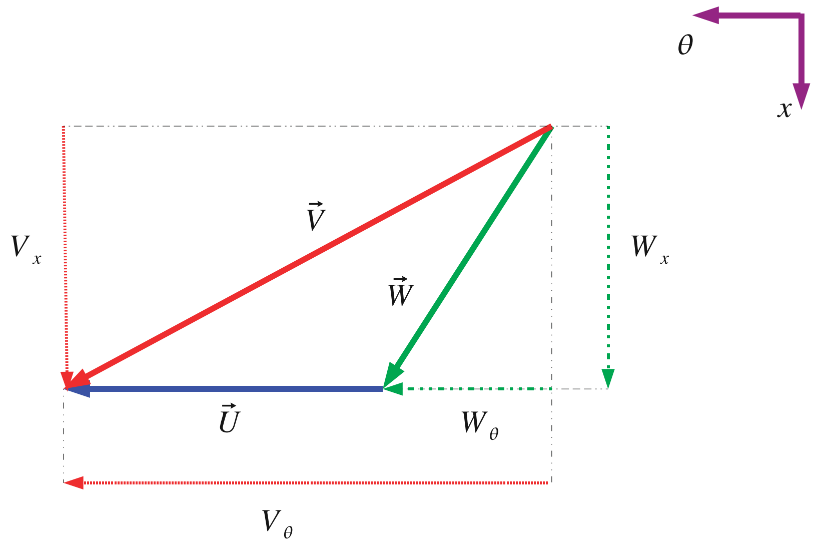

Figure 5.5: Generic Velocity Triangle

We can then substitute the two obvious relationships so:

$$V^2 = V_x^2 + V_\theta^2 = W_x^2 + (W_\theta + U)^2 = W^2 + 2W_\theta U + U^2$$

So we can express the absolute velocity magnitude V in terms of relative velocities and the blade speed:

$$V^2 = W^2 + 2W_\theta U + U^2 \tag{5.18}$$

Now consider a rotor row with entry (point 1) and exit (point 2):

$$I_1 = I_2 \implies h_{01} - UV_{1\theta} = h_{02} - UV_{2\theta}$$

expanding out the h_0 term yields:

$$h_1 + \frac{V_1^2}{2} - UV_{1\theta} = h_2 + \frac{V_2^2}{2} - UV_{2\theta}$$

Consider the left hand side of this equation

$$LHS = h_1 + \frac{V_1^2}{2} - UV_{1\theta}$$

Substitute for V using Equation 5.18 and for V_θ using $V_\theta = U + W_\theta$:

$$LHS = h_1 + \frac{W_1^2}{2} - \frac{U_1^2}{2}$$

We can simplify this further by defining another new term called the *relative stagnation enthalpy* given by:

$$h_{0rel} = h + \frac{W^2}{2} \tag{5.19}$$

Apply this to the right hand side of the equation as well as the left rothalpy conservation for a rotor row reduces to:

$$h_{0rel1} - \frac{U_1^2}{2} = h_{0rel2} - \frac{U_2^2}{2} \tag{5.20}$$

where station 1 is the entry to the rotor row and station 2 is the exit to the rotor row.

Equation 5.20 brings out two significant points about turbomachinery:

- Firstly for axial machines with little change of radius $U_1 = U_w$ so rothalpy conservation is equivalent to conservation of relative stagnation enthalpy. Compare this with the stator case where rothalpy conservation is equivalent to conservation of stagnation enthalpy. Constant relative stagnation enthalpy implies that no work is done in the relative rotor coordinate system - which may be confusing but there is no movement in this coordinate system as the observer is attached to the rotor blades - therefore in this coordinate system there is no work. The energy inputs or outputs depend on the coordinate system chosen but the pressure and temperature are independent of the coordinate system chosen.
- Secondly the U^2 term can be rewritten:

  $$\frac{U^2}{2} = \frac{(\omega r)^2}{2}$$

  and can be thought of as the potential energy of a centrifugal force field due to rotation. With the same rotational speed a higher radius object will have a higher tangential velocity.

5.4 Problems

1. An industrial gas turbine operates at an 8.8:1 pressure ratio and a mass flow of 77 kg/s. The exhaust temperature is at $437°C$ and the inlet temperature to the machine is around $1000°C$. The machine is to be designed for a constant axial velocity of 200 m/s. (This data is based on the Rolls-Royce Avon, an engine that dates from the 1950s but is still used as low efficiency high reliability engine for stationary power such as pipeline pumping.)

   For this gas turbine, draw to scale the meridional view with:

   (a) a constant mean radius of $0.4\ m$.

   (b) a constant outer diameter (to minimise cross sectional area) of $1.05\ m$.

2. Consider an axial compressor rotor blade row. The inlet velocity is 150 m/s and is in the axial direction. The blade speed is 180 m/s and the relative outlet angle is $-30\circ$ to the axial. The axial velocity is constant across the row. Calculate the relative flow angle at inlet and the absolute swirl velocity at exit and the absolute flow angle. Calculate also the specific work output. *Answers:* $-50.2°$*;* 93.4 m/s*,* $+31.9°$*,* 16.8 kJ/kg

Chapter 6

Efficiency and Reaction

6.1 Efficiency

There are over one hundred different ways of defining the efficiency of a turbomachine, though only one of them will be discussed in this book. Consider a turbine on a h-s diagram, Figure 6.1.

Three points are shown on this diagram 1 the entry condition to the machine, 3 the real exit condition and 3s the isentropic (or ideal) exit condition. (We use 3 rather than 2 as we will consider the stator/rotor interface as 2 later) Since the real exit condition will contain irreversibility's due to fluid friction, heat transfer over a finite temperature difference and so on the entropy in the real case increases meaning the line 1 to 3 is a diagonal. The ideal process is a straight line on the chart as the

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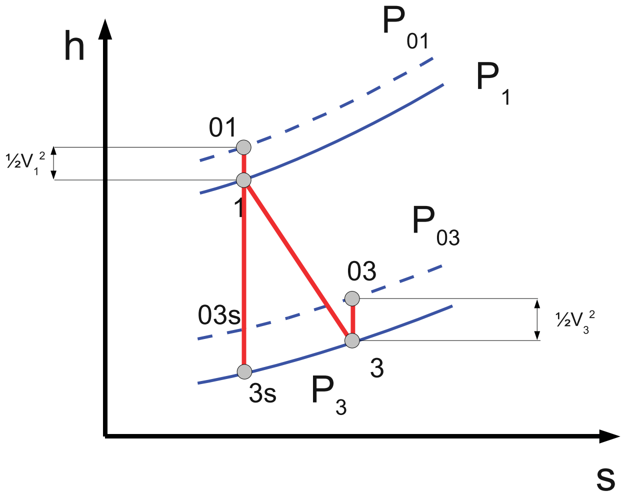

Figure 6.1: Enthalpy-Entropy Diagram for a Turbine

change in entropy is zero for an reversible process.

One of the reasons there are hundreds of definitions of efficiency is that there are various ways of accounting for the kinetic of the flow at inlet and outlet. Recall that for each location in a fluid machine there is a static condition and a corresponding stagnation condition. The stagnation condition is the condition that occurs when the fluid is brought reversibly to rest so the entropy is the same between the static and stagnation conditions. For the turbine stage on the h-s diagram (Figure 6.1) at each condition 1,3 and 3s there is a corresponding stagnation condition 01,03 and 03s which are also plotted.

Note that the difference between static and stagnation conditions is always a vertical line on the h-s chart as it is by definition an isentropic process. $h_0 = h + \frac{V^2}{2}$ by definition, so the vertical length of the line is $\frac{V^2}{2}$.

Efficiency can be defined in terms of actual work output and ideal work output. For a turbine, the Isentropic Total-to-Total Efficiency is the ratio:

$$\frac{\text{Actual Work Output}}{\text{Ideal Work Output}} = \frac{w_A}{w_I} \implies \eta_{it} = \frac{h_{03} - h_{01}}{h_{03s} - h_{01}} \tag{6.1}$$

The ideal work output is always greater than the actual work output, the real (non-deal) process serves to reduce the power output of the device. Note that the work output will be negative which is a matter of sign convention, if the work output for turbines is negative the work input for compressors is positive - one has to pick one to be negative! Often however designers deal with only turbines and make the work output positive to avoid writing minus signs in front of every calculation.

Figure 6.2 shows the h-s diagram for a compressor or pump. Since compressors deliver energy to the fluid the net result is a rise in pressure. For a compressor the ideal work is always less than the actual. The isentropic total to total efficiency for a compressor is given by:

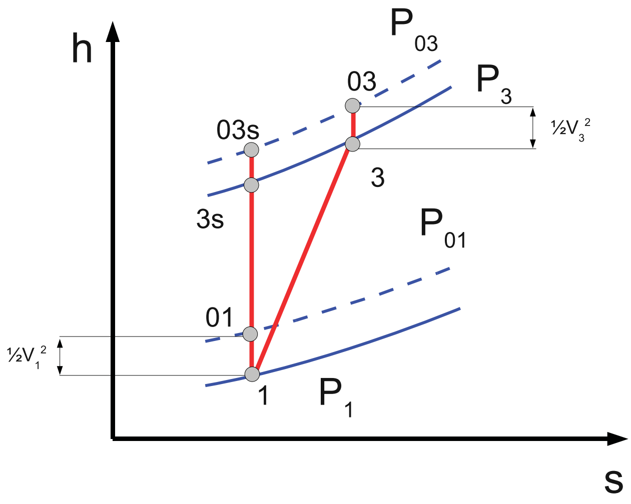

Figure 6.2: Enthalpy-Entropy Diagram for a Compressor

$$\frac{\text{Ideal Work Input}}{\text{Actual Work Input}} = \frac{w_I}{w_A} \implies \eta_{ic} = \frac{h_{03s} - h_{01}}{h_{03} - h_{01}} \tag{6.2}$$

Example A compressor stage working on air at 1 bar and $25°C$ has a work input of 17 kJ/kg and an isentropic efficiency of 0.9. The velocity at inlet and exit is the same. Assuming that the air properties are unchanged over the stage calculate the pressure output of the stage.

Solution The work input is the specific work input so and since the velocity at inlet and exit are the same $V_3 = V_1$ the difference between static and stagnation enthalpies is the same.

$$w_A = h_3 - h_1 = 17\ kJ/kg$$

Since we can approximate air as a perfect gas ($C_p = 1.01\ kJ/kgK$):

$$Cp\Delta T = \Delta h \implies \Delta T = \frac{\Delta h}{C_P} \implies T_3 - T_1 = \frac{17}{1.01} = 16.83\ K$$

The efficiency for a compressor using a perfect gas can be expressed in terms of temperature rises:

$$\eta = \frac{h_{3s} - h_1}{h_3 - h_1} = \frac{T_{3s} - T_1}{T_3 - T_1} = 0.9 \implies T_{3s} - T_1 = 0.9(T_3 - T_1) = 15.15\ K$$

From basic thermodynamics we recall that for an isentropic compression:

$$\frac{T_{3s}}{T_1} = \left(\frac{p_3}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

Now $\gamma \approx 1.4$ for air at $20°$ so:

$$\frac{p_3}{p_1} = \left(\frac{298 + 15.15}{298}\right)^{\frac{1.4}{0.4}} = 1.19\ bar$$

Since the inlet is at atmospheric pressure 1 bar the pressure output from the stage is 1.19 bar.

6.1.1 Using Efficiency

Actual efficiencies can be determined by either measurement or computationally intensive simulations of the fluid flow through a turbomachine. For the purposes of this book they will simply be based on given values. Isentropic efficiencies of over 90% have been reported for turbines and compressors are typically a few percent less efficient.

The key point is that efficiency provides a link between the work from flow geometry (Euler work equation) and the actual enthalpy or pressure change in the fluid.

- For a steam turbine the ideal work is given by: $w_I = h_{01} - h_{03s}$. Finding the ideal work involves determining the enthalpies at inlet and exit. At inlet h_1 is determined from the h-s diagram or steam tables given inlet pressure and temperature. The exit condition h_{3s} is derived from an isentropic expansion (vertical line on the h-s chart) and a known exit pressure.
- For a gas turbine the ideal work is obtained using the ideal gas laws:

$$w_I = h_{01} - h_{3s} = C_p T_{01}\left(1 - \frac{T_{03}}{T_{01}}\right) = C_P T_{01}\left(1 - \left(\frac{p_{03}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}}\right)$$

- For a pump operating on an incompressible fluid (water or low speed gas flow) then the ideal work can also be obtained from an equation:

$$w_I = h_{01} - h_{03s} = \frac{\Delta p}{\rho} = g\Delta H_0$$

where ΔH_0 is the change in total head across the turbine.

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Example A steam turbine stage operates with inlet conditions of $120 bar$ and $500°$, the exit pressure is $10\ bar$. If the efficiency of the stage is 90% calculate the specific work output.

Solution The strategy here is to look up h_1 and h_{3s} from a suitable set of Steam Tables such as Rogers and Mayhew (1994) and then use the efficiency to get the actual work out.

From steam tables or a Mollier Chart:

$$h_1 : 120\ bar, 500°C = 3350\ kJ/kg$$

Sketch a line of constant entropy on the Mollier chart from point 1 to the $10\ bar$ line or interpolate using tables to determine h_{3s}:

$$h_{3s} = 2730\ kJ/kg$$

rearranging the definition of turbine efficiency (Equation 6.1) the actual work is:

$$w_A = \eta_{it}(h_{3s} - h_1) = 0.9 \times (2730 - 3350) = -558\ kJ/kg$$

The answer is negative as our sign convention is that energy transfers out of the system are negative, but generally such sign conventions are not important in turbomachinery as one normally knows whether the device is supposed to extract or deliver energy!

6.1.2 Other Efficiency Definitions

In this book only the total to total efficiency (using only h_0) in definitions of efficiency is used but using static or stagnation conditions in the definitions of efficiency allows the kinetic energy of the leaving fluid to be used in different ways:

1. Where the exit kinetic energy is useful for example for thrust in a jet engine, or subsequent stages in a multi-stage machine. In this case the energy is not part of the maximum work available.
2. Where the exit kinetic energy is not useful for example in the last stage of a steam turbine exhausting to the condenser, or a hydraulic turbine where the water flows away done the exit pipe. An ideal turbine would make use of that exit kinetic energy so it must be included in the definition of the maximum work available.

These different cases explain why other definitions of efficiency may be used.

6.2 Reaction

There are three difficult ideas presented in this book. The cascade view, velocity triangles and the concept of reaction. Once these three concepts have been mastered they open the door to a much wider understanding of turbomachinery which is why this book concentrates on them. The third concept to master is that of Reaction which is defined as the ratio of the static enthalpy drop through the rotor divided by the static enthalpy drop through the stage. Or in equation form:

$$R = \frac{\Delta h_{ROTOR}}{\Delta h_{STAGE}}$$

Recall from basic thermodynamics that:

$$Tds = dh - \frac{1}{\rho}dp = 0$$

If the process is isentropic $ds = 0$ then:

$$dh = \frac{1}{\rho}dp \implies \Delta p \approx \frac{1}{\rho}\Delta h$$

Most turbomachines are quite efficient so approximating their operation as isentropic is reasonable. This means that enthalpy drops can be approximated as pressure drops and gives the widely used definition of Reaction:

$$R = \frac{\Delta h_{ROTOR}}{\Delta h_{STAGE}} \approx \frac{\Delta p_{ROTOR}}{\Delta p_{STAGE}} \tag{6.3}$$

Consider the turbine shown in Figure 2.4 on page 25 and in the context of Reaction a number of questions arise: How should we assign the pressure drop of the stage between the stator and the rotor? What blade angles do we need to set to obtain the desired pressure drops? What levels of reaction are desirable? These are questions that we will answer next and in Chapter 8.

6.3 Reaction on the $h - s$ Diagram

It is possible to show the level of reaction of a particular turbomachine on the h-s chart and get a good understanding of what is going inside a turbomachine as a result. This can get somewhat complex so the chart is built up in stages. Firstly consider Figure 6.3 which shows a real process in a turbine expanding from pressure p_1 to pressure p_3. The geometry over which this takes place can be seen in meridional and cascade view in Figure 2.4 on page 25. So station 1 is the stator inlet, station 2 is the rotor/stator interface and station 3 is the rotor exit. The intermediate pressure p_2 is shown in Figure 6.3 along with the static and stagnation conditions at 1 and 3.

We now consider station 2 on the h-s diagram in more detail. For the stator 1 to 2 $h_{01} = h_{02}$ because no work is done in the stator we can then add a point 02 to the h-s diagram showing this. Since the difference between stagnation and static enthalpy is given by $\frac{V^2}{2}$ we can draw the corresponding velocity magnitudes on the h-s chart for V_1,V_2 and V_3. This done in Figure 6.4.

Now since the stagnation enthalpy is constant:

$$h_{01} = h_{02} = h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \implies h_1 - h_2 = \frac{V_2^2}{2} - \frac{V_1^2}{2}$$

From Figure 6.4: $V_2 > V_1$ so there is a static enthalpy drop in the stator. For a turbine the flow is accelerated in the stator.

The level of static enthalpy or pressure drop is controlled by the velocity V_2 for high values of V_2 the pressure drop is greater. V_2 is set by turbomachine geometry so we can *control the level of reaction in the turbine by altering the blade angles and blade height through the machine.*

Finally we consider a turbine where the mean radius is the same so the blade speed U is the same at entry to and exit from the rotor. This restricts the generality of our diagram somewhat but this condition is true of a great deal of axial flow turbines and provides some useful insights. If the blade speed is the same through the turbine the relative stagnation enthalpy will be constant through the rotor blade (See Chapter 5). Two new points are added to the h-s diagram 03R and 02R which represent the relative stagnation enthalpy at points 2 and 3 respectively, this is shown in Figure 6.5. Points 03R and 02R lie above the corresponding static conditions 2 and 3 and the distance between them is given by $\frac{W^2}{2}$ from the definition of relative stagnation enthalpy. Figure 6.5 shows that for an axial flow turbine the flow is accelerated through a turbine $W_3 > W_2$.

Reaction is often a cause of some confusion for students. This is because intuitively one might expect the power output from a stage to depend on the pressure drop across it - but as we showed with Equation 5.12 the work input or output from a turbomachine depends entirely on the flow turning the rotor row. The relationship between pressure drop and velocity change can be seen in the enthalpy-entropy diagram in Figure 6.5.

The effect of the real process irreversibilities can also be seen in Figure 6.5, an ideal process is a straight line ($\Delta s = 0$). So if the process was ideal it could be drawn on the h-s diagram using a vertical line from 01 to the pressure p_2, the velocity V_2 would then be larger than in the real ($\Delta s > 0$) case.

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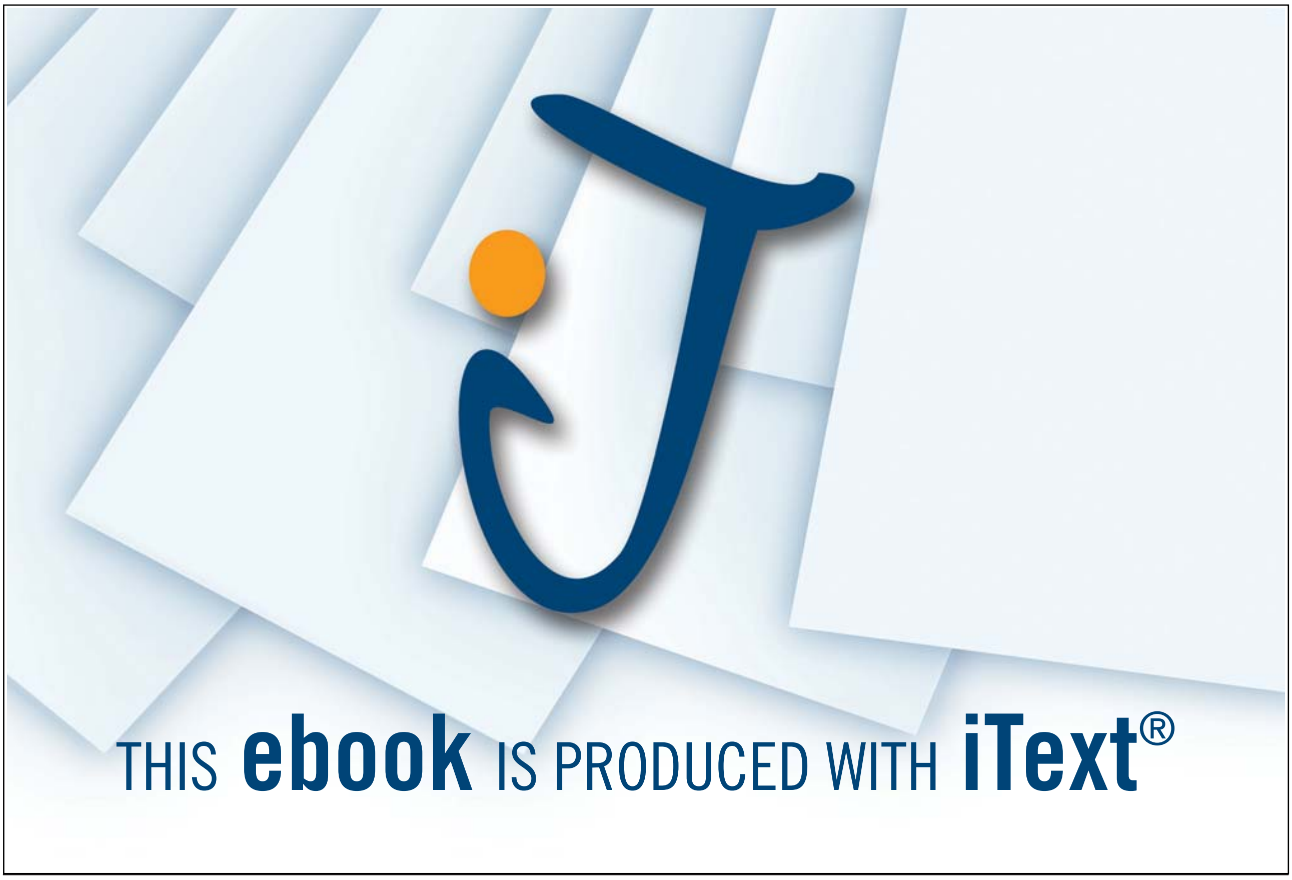


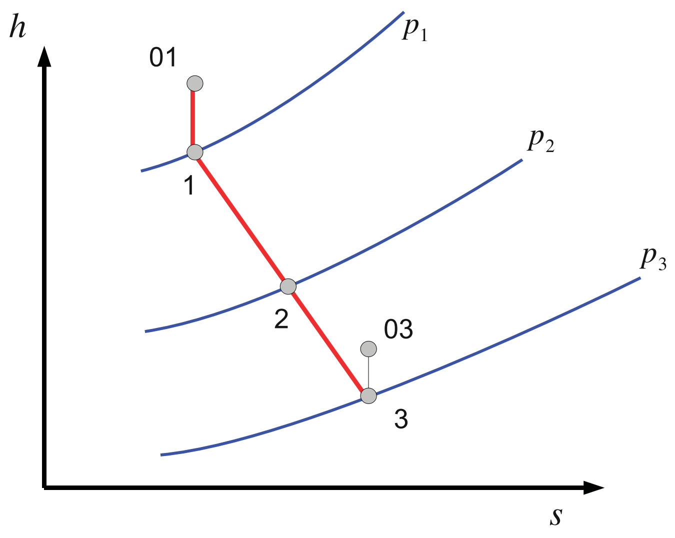
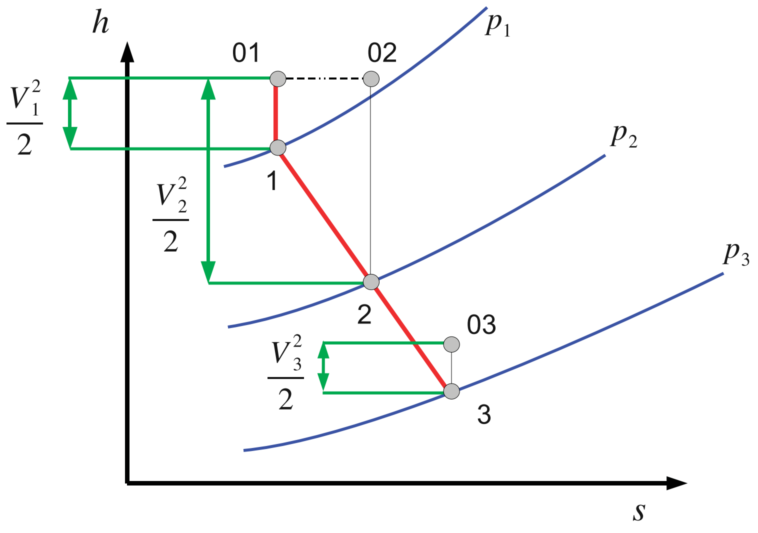

h
01
p_1
1
p_2
2
03
p_3
3
s

Figure 6.3: Basic h-s diagram


h
01
02
p_1
$\frac{V_1^2}{2}$
1
$\frac{V_2^2}{2}$
p_2
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p_3
03
$\frac{V_3^2}{2}$
3
s

Figure 6.4: h-s diagram with h_0

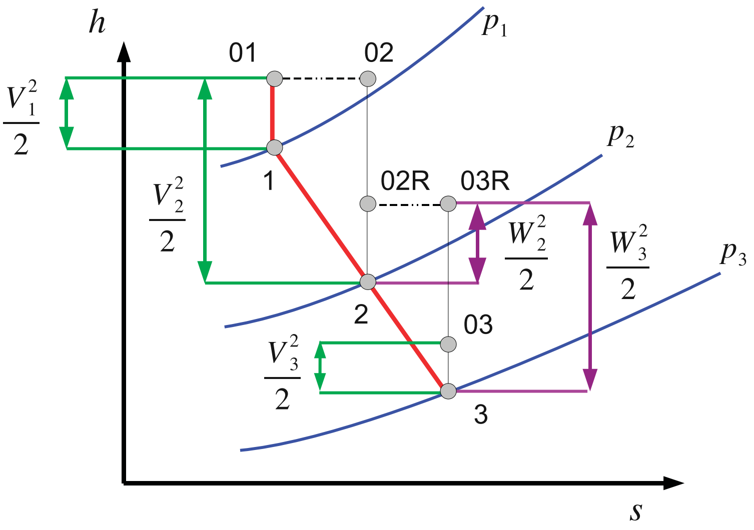

Figure 6.5: h-s diagram with h_0 and h_{0rel}

6.4 Problems

Note that some of these problems use the same data as previous chapter problems so if you have kept your answers you may save some time.

1. An axial compressor has a rotor blade row where the inlet velocity is $150\ m/s$ and is in the axial direction. The blade speed is $180\ m/s$ and the relative outlet angle is $-30°$ to the axial. The axial velocity is constant across the row.

   These rotor blades are followed by a set of stator blades to form a complete stage. The stators turn the flow back to the axial direction and the axial velocity is constant across the whole stage. If the total-to-total efficiency is 90%, calculate the total pressure at outlet from the stage. Assume air properties with inlet stagnation conditions of 20° and $1.0\ bar$. *Answer:* $1.187\ bar$

2. Calculate the reaction of the stage in the previous question. *Answer: 0.74*

3. The flow at exit from a turbine stator row has a velocity of $100\ m/s$ at an angle (α_2) of $70°$ to the axial direction. The rotor row is moving with a velocity of $50\ m/s$. At exit from the rotor row the relative flow angle (β_3) is $-60°$. The axial velocity is constant across the row. Calculate the power output for a flow rate of $4\,kg/s$. Calculate the total pressure drop across the stage if the efficiency is 90 % and the fluid density is constant at $1.2\,kg/m^3$. Calculate also the static pressure drop. What is the stage reaction? *Answers:* $20.6\,kW$, $6.88\,kPa$, $6.93\,kPa$,0.154

Chapter 7

Dimensionless Parameters for Turbomachinery

Dimensionless coefficients are a very useful technique to bring out trends between groups of variables in fluid mechanics. Further details can be found in basic fluid mechanics texts Massey (1989) but essentially the technique works by constructing physically plausible relationships between variables. Consider the relationship between static pressure, dynamic head and stagnation pressure for example:

$$p_o = p + \frac{\rho V^2}{2}$$

For this relationship to be physically possible ρV^2 must have the same units as pressure, in simple

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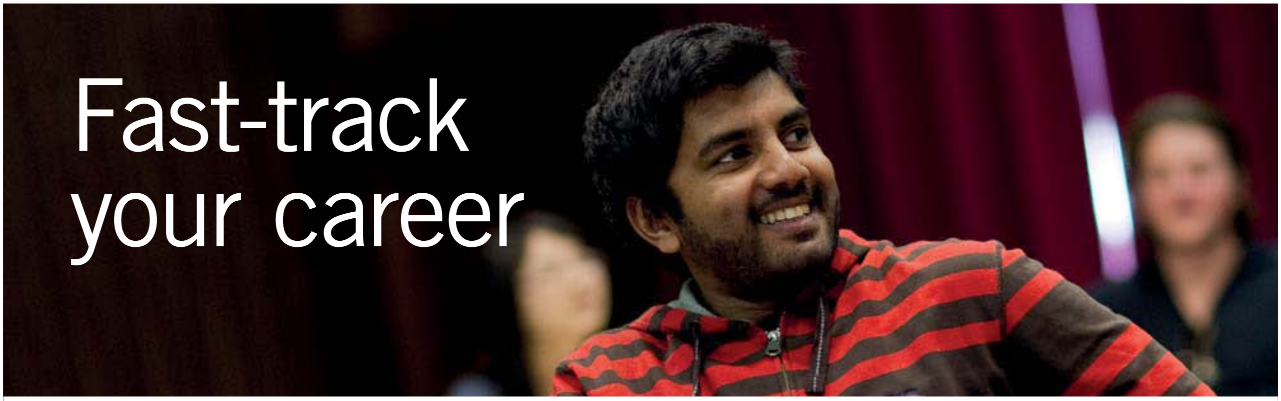

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terms one cannot count two oranges and expect to end up with an apple!

Pressure can be explained as a force per unit area, force can be expressed as a mass times an acceleration so the units of pressure will be given by:

$$\frac{mass \times acceleration}{area} = \frac{mass \times \frac{length}{time^2}}{length^2} = \frac{mass}{time^2 \times length}$$

The units of ρV^2 can similarly be expressed:

$$density \times velocity = \frac{mass}{length^3}\frac{length^2}{time^2} = \frac{mass}{time^2 \times length}$$

Therefore the equation is said to be dimensionally correct. Using this principle one can construct a number of dimensionless parameters or dimensionless coefficients. For example reaction is a dimensionless coefficient as the units on the top of the expression cancel the units on the bottom of the expression.

Dimensionless parameters provide a number of advantages:

1. With an appropriate choice of dimensionless parameters the performance of machines can be characterised using only a few key variables.
2. Given data on one size of machine performance can be predicted at different sizes.
3. Given data on one set of operating conditions behaviour at different operating conditions can be predicted
4. Enable designers to pick a particular machine shape of maximum efficiency.

Which dimensionless parameters are important depends very much on the application area several of which are discussed in turn.

7.1 Coefficients for Axial Machines

For axial flow turbines there are four dimensionless coefficients that have been found to be important. The first of these is the *flow coefficient*:

$$\phi = \frac{V_x}{U_m} \tag{7.1}$$

which is the axial velocity divided by the mean blade speed. For a given blade speed and height this gives some idea of the flow through the machine.

The second of these is the *stage loading coefficient* or *work coefficient*:

$$\Psi = \frac{w_x}{U_m^2} = \frac{\Delta h_o}{U_m^2} \tag{7.2}$$

which is a measure of the work done in a stage. The other two important coefficients are the efficiency η and the reaction R.

Case	Flow Coefficient	Stage Loading Coefficient
Aircraft Engine Compressor	0.4 to 0.70	0.35 to 0.50
Aircraft Engine HP Turbine	0.5 to 0.65	1.0 to 2.0
Aircraft Engine LP Turbine	0.9 to 1.0	1.0 to 2.0

Table 7.1: Typical Values of Ψ and Φ

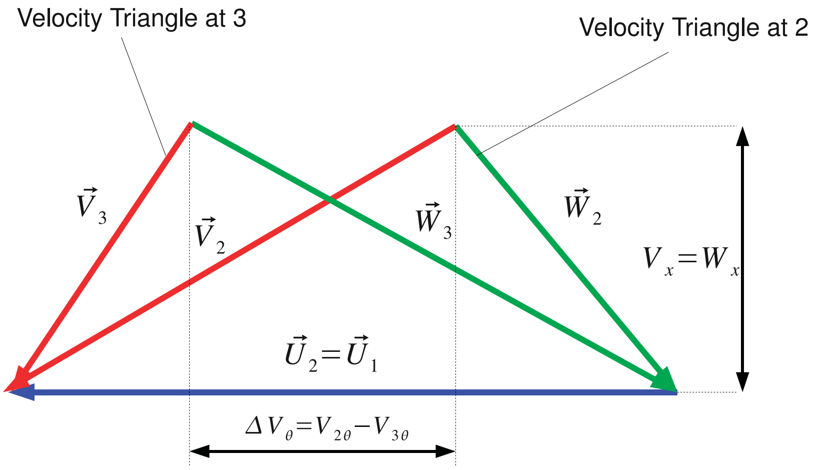

Figure 7.1: Velocity triangles for exit and inlet combined

In general the designer of turbomachinery blading does not have complete freedom to determine the dimensionless coefficients that the design will operate at. For example the mean blade speed is often fixed by mechanical considerations, or the rotation frequency of the machine is fixed by the frequency of the electrical supply and the number of poles in the electrical generator. The stage loading is often fixed by the number of stages and the required enthalpy drop in the machine.

Table 7.1 shows some typical values of the stage loading and flow coefficients for aircraft engines. Note the much lower loading for compressors compared to turbines, the reason for this is that compressors operate in an adverse pressure gradient. That is compressors have a higher pressure at exit than they do at inlet and this makes the boundary layers along the blade surfaces much more likely to separate. The aerodynamic design is then much more challenging than for a turbine so lower levels of loading are applied.

For an axial flow turbine the mean blade speed is usually constant across a blade row as there are no large radius changes in most axial flow turbines. It is impossible to change the blade speed ω *within* a blade row. It is therefore often convenient to plot the exit velocity triangle and the inlet velocity triangle on top of one another (Figure 7.1.

Now the Euler equation (Equation 5.12) gives:

$$w = \omega(V_{2\theta} r_2 - V_{1\theta} r_1) = U_m \Delta V_\theta$$

So substituting this into the expression for work coefficient:

$$\Psi = \frac{w}{U_m^2} = \frac{1}{U_m}\Delta V_\theta$$

and remembering the definition of flow coefficient:

$$\phi = \frac{V_x}{U_m}$$

Two things become apparent:

- For a given blade speed U_m: Ψ gives the width of the diagram
- For constant axial velocity V_x: ϕ gives the height of the diagram

The representation in Figure 7.1 is compact and is often used when designing axial flow machines, especially when large numbers of stages have to be put together - in this book the emphasis is simplicity not compactness so it is not used very much.

One of the key points to take away from Figure 7.1 is that given a constant blade speed and axial velocity changing the dimensionless coefficients has a direct bearing on the flow angles. So for the triangles shown in Figure 7.1 if the stage loading coefficient is reduced for example but the flow coefficient and blade speed kept the same, so U and V_x are constant then the absolute flow angles α_1 and α_3 will reduce. There is a direct link between machine geometry and dimensionless coefficients.

Finally an important simplification with constant blade speed is that since $V_\theta = U_m + W_\theta$ then $\Delta V_\theta = \Delta W_\theta$

Example Given a turbine blade row with constant axial velocity of $150 m/s$ at $5000 rpm$ on a mean radius of $0.7\ m$ and an absolute flow angle at exit from the stator of $70°$. The turbine operates with axial leaving flow and is a repeating stage. Calculate the flow coefficient, stage loading coefficient and reaction.

Solution This problem is a straightforward application of dimensionless coefficients. First calculate the mean blade speed:

$$U_m = \omega r = 5000 \times \frac{2\pi}{60} \times 0.7 = 366.5\ m/s$$

$$\phi = \frac{V_x}{U_m} = \frac{150}{366.5} = 0.409$$

$$\psi = \frac{w}{U_m^2} = \frac{U_m(V_{3\theta} - V_{2\theta})}{U_m^2} = \frac{V_{3\theta} - V_{2\theta}}{U_m}$$

Now the turbine row is axial leaving so $V_{3\theta} = 0$ from the generic velocity triangle:

$$V_{2\theta} = V_x \tan\alpha_2 = 150 \tan 70 = 412.12\ m/s$$

so the stage loading coefficient is:

$$\psi = \frac{412.12}{366.5} = 1.124$$

Reaction is given by $R = \Delta h_{ROTOR}/\Delta h_{STAGE}$. The stage enthalpy drop is given by the work output as the turbine has repeating stages so $V_1 = V_3$.

$$\Delta h_{STAGE} = \Delta h_0 = w = U_m(\Delta V_\theta) = 366.5(412.12) = 151.04\, kJ/kg$$

From Figure 6.5 we see that the static enthalpy drop across the rotor is given by:

$$\Delta h_{ROTOR} = \frac{W_3^2 - W_2^2}{2} = \frac{W_{3\theta}^2 - W_{2\theta}^2}{2}$$

Since the turbine axial velocity is constant $W_{2x} = W_{3x}$ and so:

$$\Delta h_{ROTOR} = \frac{W_{3\theta}^2 - W_{2\theta}^2}{2}$$

However since we have constant blade speed $\Delta W_\theta = \Delta V_\theta$ so:

$$\Delta h_{ROTOR} = \frac{412.12^2 + 0^2}{2} = 84.92\, kJ/kg$$

Which means that the reaction is given by $R = 84.92/151.04 = 0.56$

7.2 Coefficients for Wind Turbines

For wind turbines two coefficients are in widespread use, the first C_P or power coefficient is essentially an efficiency by a different name:

$$C_P = \frac{\text{Power from Turbine}}{\text{Power in Wind}} = \frac{P}{\frac{1}{2}\rho V^3 A} \tag{7.3}$$

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where P is the power output, V is the velocity of the wind and A is the swept area of the machine. The second coefficient is the tip speed ratio, which is the ratio of the wind speed to the tangential velocity of the tip of the turbine:

$$\lambda = \frac{\omega r_{tip}}{V} \tag{7.4}$$

Performance of wind turbines is usually presented as C_P vs λ curve, this is often called a "power curve". A power curve for a Nordex N80 turbine [1]which produces 2.5MW at full power is shown in Figure 7.2. The power curve shows a characteristic of many turbomachinery devices, that there is distinct peak in efficiency with a lower device effectiveness away from the design point.

Example Given the power curve shown in Figure 7.2. Calculate the power output of the device at 10 rpm in a 10 m/s wind given that the turbine is 80m in diameter and the density of air can be taken as $1.15 kg/m^3$.

Solution This problem reduces to finding the tip speed ratio λ looking up the power coefficient on the graph and then calculating the power output. The tip speed ratio λ gives:

$$\lambda = \frac{\omega r_{tip}}{V} = \frac{\frac{10\times 2\pi}{60} \times \frac{80}{2}}{4} = 10.47$$

So look up the power coefficient that corresponds to 10.47 on Figure 7.2 which is approximately $C_P = 0.37$. The area of the turbine is given by $A = \pi r_{tip}^2 = \pi \times 40^2 = 5026\, m^2$, the assumption is that the area of the nacelle is negligible. Putting this all together:

$$C_P = \frac{P}{\frac{1}{2}\rho V^3 A} \implies P = C_P \frac{1}{2}\rho V^3 A = 0.37 \times \frac{1}{2} \times 1.15 \times 10^3 \times 5026 = 1.069\, MW$$

7.3 Coefficients for Hydraulic Machines

Hydraulic machines are those turbomachines that operate with liquid (most often water) as the working fluid. These are enormously significant devices which produce up to 20% of the worlds electricity in large and small hydroelectric plants. Pumps are vital to the infrastructure of water and fuel supply.

Dimensional analysis of hydraulic machines yields a different set of coefficients, these are the *flow coefficient*:

$$\Pi_1 = \frac{Q}{ND^3} \tag{7.5}$$

where N is the number of revolutions per second made by the machine and D is the machine diameter. We give the coefficients the symbol Π. The next dimensionless parameter is the *head coefficient*:

$$\Pi_2 = \frac{gH}{N^2 D^2} \tag{7.6}$$

where H is the head produced or absorbed by the hydraulic machine and g is the acceleration due to gravity. Finally the power coefficient is given by:

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[1] `http://www.nordex-online.com/en/products-services/wind-turbines/n80-25-mw/`

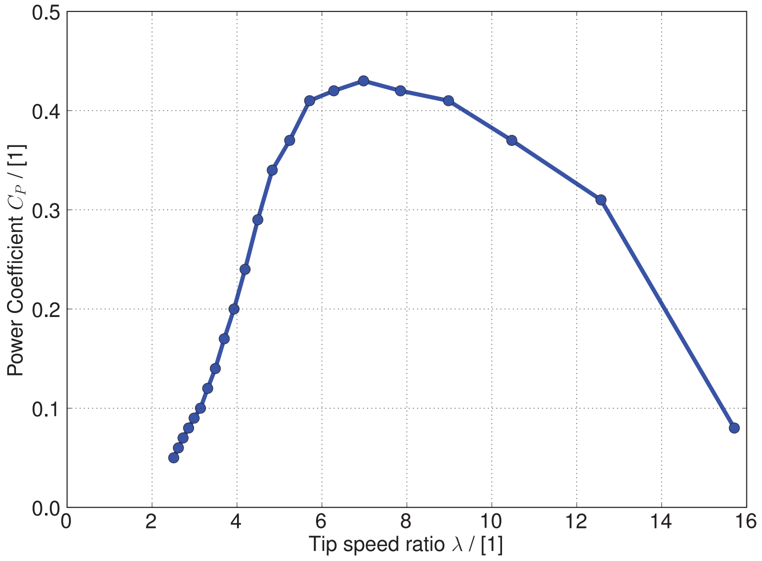

Figure 7.2: C_P vs λ for 2.5MW Wind Turbine

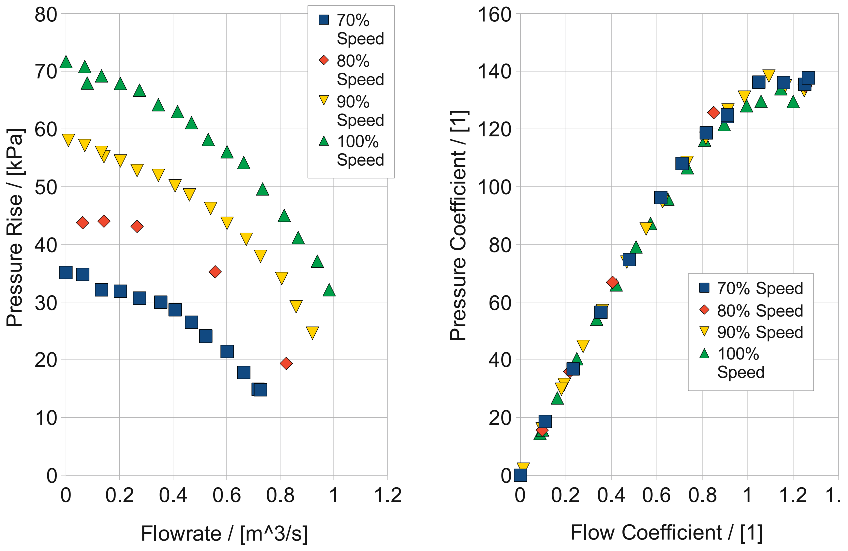

Figure 7.3: Collapsing Pump Data onto Non-dimensional Curves

$$\Pi_3 = \frac{P}{\rho N^3 D^5} \tag{7.7}$$

The power of dimensional analysis can be seen in Figure 7.3 the left hand plot consists of pressure rise against flow in the pump at different % of the maximum speed and is taken from a pump experiment at the University of Durham. The right hand plot presents the same information in non-dimensional terms this time using flow coefficient $\frac{Q}{ND^3}$ compared to pressure rise coefficient $\frac{P}{\rho N^3 D^5}$ the data is exactly the same but reduces the relationship at different speeds to a single line. This shows how plotting parameters in terms of dimensionless quantities is a powerful method for reducing the complexity and amount of information an engineer needs to understand.

Clearly efficiency is a non-dimensional parameter for hydraulic machines and again it varies between pumps and turbines as follows:

$$\eta_{pump} = \frac{\text{Ideal Work}}{\text{Actual Work}}, \eta_{turbine} = \frac{\text{Actual Work}}{\text{Ideal Work}}$$

7.3.1 Specific Speed for Turbines

Note that for a turbine:

$$\frac{\Pi_3}{\Pi_1 \Pi_2} = \frac{P}{\rho Q g H} = \eta$$

For a pump the reciprocal applies. So if for a model turbine we obtain test data and plot the dimensional parameters against each other the performance of larger machines of the same type can be predicted. For a particular machine the operating conditions are expressed by values of N, P and H. The rotational speed N is normally fixed by the frequency of electricity supply, P and H are set by the flow rate and the height drop of the physical location that the hydro-electric scheme is proposed for. It would therefore be useful to have a dimensional group which includes N, P and H but not D so that it would be independent of the machine size. This can be obtained by manipulating the dimensionless groups for a turbine to obtain a new dimensionless coefficient:

$$\Pi_4 = \frac{\Pi_3^{1/2}}{\Pi_2^{5/4}} = \frac{NP^{1/2}}{\rho^{1/2}(gH)^{5/4}}$$

The point of machine design is to obtain maximum efficiency and there is generally only one set of values of Π_1 and Π_2 and Π_3 for which this occurs. For a given shape of machine we are therefore interested in a unique value of Π_4. We call this dimensionless specific speed:

$$K_s = \frac{\Pi_3^{1/2}}{\Pi_2^{5/4}} = \frac{NP^{1/2}}{\rho^{1/2}(gH)^{5/4}} \tag{7.8}$$

K_s has a unique value for maximum efficiency for a given shape of machine. It is a *shape parameter* independent of the size, D. By calculating the value of K_s from the design specifications, we can determine the most efficient machine type.

Normally hydraulic turbines use water and remain on the surface of the earth so ρ and g are dropped to give a dimensional specific speed:

$$N_S = \frac{NP^{1/2}}{H^{5/4}} \tag{7.9}$$

For N in rev/min, P in kW and H in m, $N_S = 1042K_s$.

The term specific speed arises from the idea of a given type of machine producing unit power (1kW) at unit head (1m), N_s is the speed it would run at in rev/min. It is *much better* to think of it as a shape parameter.

For commercial machines there are size limitations the upper limit for N is determined by material strength (centrifugal forces) and machines with a small power output P of a given type become less efficient - giving a range of applicability for different types of hydraulic machine.

7.3.2 Specific Speed for Pumps

For a pump the requirement is more likely to be flow Q, rather than power so a different dimensionless group is picked.

$$\Pi_5 = \frac{\Pi_1^{1/3}}{\Pi_2^{1/2}} = \frac{Q^{1/3} N^{2/3}}{(gH)^{1/2}}$$

raise to the power 2/3 to get:

$$K_{s'} = \frac{NQ^{1/2}}{(gH)^{3/4}} \tag{7.10}$$

which is the dimensionless specific speed for a pump, which again has a unique value for maximum efficiency. As for turbines there is also a dimensional specific speed:

$$N_S = \frac{NQ^{1/2}}{H^{3/4}} \tag{7.11}$$

For N in rev/min, Q in m/s and H in m, $N_s = 333K_{s'}$

Note that the specific speed for pumps and turbines are related by a simple ratio - they are fundamentally the same concept just applied to different machines.

7.3.3 Using Specific Speeds

Pump and turbine manufacturers will provide charts and diagrams that relate pump or turbine geometry to specific speed. These charts are based on the manufacturer's experience with particular machines as the choice of machine type depends on the maximum efficiency which can only be determined by testing. Note that different companies or organisations have widely different choices for the units they apply to their charts - be sure to use the same units as the person who made the chart! .

Figure 7.4 shows a plot of specific speed against head from a large number of hydroelectric power plants produced by Alstom Power from 1965-2006. The units of specific speed are actually $kg^{1/2}m^{7/5}s^{-5/2}$ or in other words not very meaningful so they are not recorded on the graph. The graph shows clearly the expected trend of specific speed vs head. This can also be obtained from Equation 7.11 where $N_s \propto H^{-1.25}$ or approximately $1/H$ in order. The point of the graph is to show how the different turbine types are grouped together.[2]

From this illustration of industry practise one can draw up some guidelines for turbine type based on specific speed, these broad guidelines are shown in Table 7.2.

---

[2] The outlier on the graph with a head of 10m and a specific speed of around 450 is actually a tidal stream turbine at La Rance in France so operates with the flow going in either direction so the design is unusual.

Case	Specific Speed
Pelton Wheel	50 and below
Francis Turbine	100 to 300
Kaplan Turbine	400 and above

Table 7.2: Values of N_s for Different Turbine Types.

Example A turbine is to be designed for a site with $400\,m$ of head an an expected power of $1\,MW$, the turbine will feed electricity into a 50 Hz electrical grid. Using specific speed estimate which sort of turbine design should be investigated further.

Solution The strategy here is to calculate the specific speed. The head and power are given so we need to find possible values or rotational speed. Electrical generators typically run at fixed multiples of turbine speed the fastest being a two pole generator which runs at the frequency of the grid, there is no limit to the number of pole pairs that can be used in the generator but up to ten pole pairs will be examined.

With a single pole pair, the rotational speed will be $N = 50 \times 60 = 3000\,rev/min$:

$$N_S = \frac{NP^{1/2}}{H^{5/4}} = \frac{3000 \times 1000^{1/2}}{400^{5/4}} = 53.03$$

Note that power is input in kW, rotational speed in rev/min and head in m. With ten pole pairs, the machine rotational speed will be $N = 300\,rev/min$ and so:

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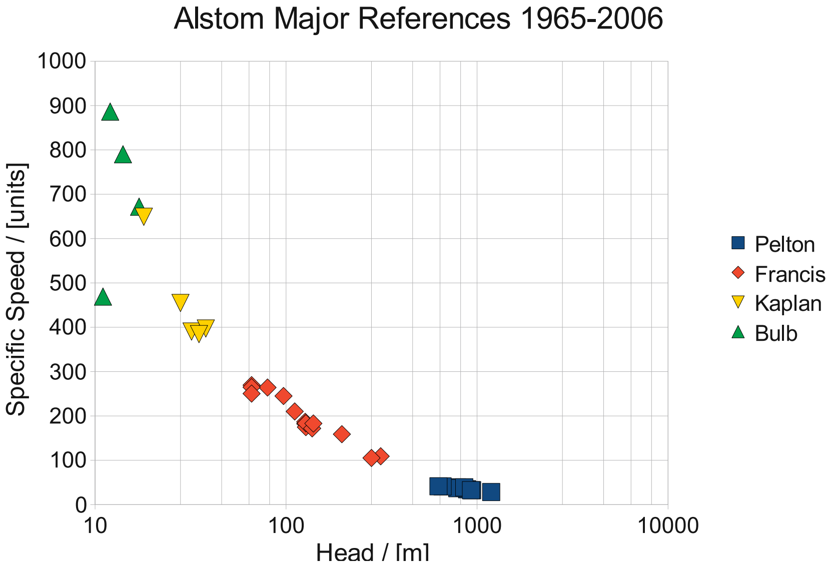

Figure 7.4: Specific Speed for a Number of Hydraulic Turbines

$$N_S = \frac{NP^{1/2}}{H^{5/4}} = \frac{300 \times 1000^{1/2}}{400^{5/4}} = 5.3$$

Comparing the values of specific speed to Table 7.2 or Figure 7.4 it is evident that a Pelton Wheel design is required. The exact choice of generator speed being determined by various design trade-offs such as between generator cost (increasing with number of poles) and windage losses (reducing with number of poles).

7.4 Problems

1. An axial flow gas turbine has three stages of compression and one stage for the turbine. The compressor operates with an axial inlet flow velocity of 250 m/s at 3000 rpm. The rotor relative exit angle is set at $-34°$ at a mean radius of $0.75\ m$. Draw the velocity triangle for the first stage rotor exit and hence estimate the specific work output. *Answer:* $15.8\ kJ/kg$
2. The turbine from the previous questions operates with an axial inlet velocity of 300 m/s. The axial velocity is kept constant through the stage. The stator blades have an absolute exit angle of $60°$ and the rotor blades have a relative exit angle of $-30°$. Calculate the specific work output and calculate the stage loading coefficient for the turbine at a mean radius of $0.75\ m$. *Answers:* $w = 107.7\ kJ/kg$, $\psi_{turbine} = 1.94$
3. Compare the stage loading coefficient of the turbine and the compressor in the previous two questions. Why is the turbine stage loading coefficient higher than that of the compressor? What limits compressor loading? *Answer:* $\psi_{comp} = 0.28$
4. The Sellrain-Silz power station in Austria has two turbines each delivering 260 MW of power. The operating head is 1233 m and the rotational speed is 500 rev/min. Show that a multiple nozzle Pelton turbine is best suited for this application.

Chapter 8

Axial Flow Machines

8.1 Reaction for Repeating Stage

The aim of this section is to relate reaction to blade dimensions for a common special case: a repeating stage with constant axial velocity. This shows how the static enthalpy drop (and therefore also the static pressure drop) are controlled entirely by the blade geometry.

Consider the h-s diagram in Figure 8.1. First consider the enthalpy drop from the stage as a whole:

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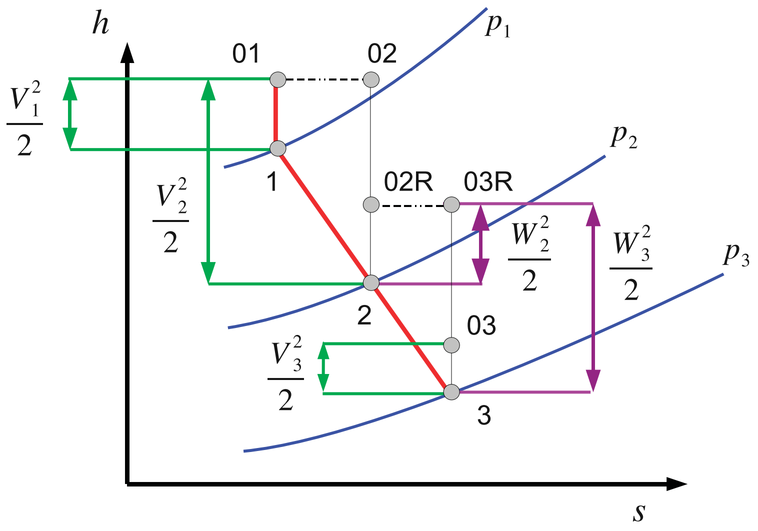


Figure 8.1: h-s diagram with h_0 and h_{0rel}

$$\Delta h_{stage} = h_1 - h_3 = \left(h_{01} - \frac{V_1^2}{2}\right) - \left(h_{03} - \frac{V_3^2}{2}\right)$$

And this case is for a repeating stage so $V_1 = V_3$:

$$\Delta h_{stage} = \Delta h_0 = w = U_m(V_{2\theta} - V_{3\theta}) = U_m(W_{2\theta} - W_{3\theta})$$

Remember that $V_\theta = \omega r + W_\theta$ if the algebraic manipulation is not obvious.

Now consider the rotor: $\Delta h_{rotor} = h_2 - h_3$ Earlier (Chapter 6) Rothalpy, I was defined:

$$I = h_0 - U_m V_\theta = h + \frac{W^2}{2} - \frac{U_m^2}{2}$$

This was shown to be constant across a blade row, so:

$$h_2 + \frac{W_2^2}{2} - \frac{U_m^2}{2} = h_3 + \frac{W_2^2}{2} - U_m^2$$

Rearranging:

$$h_2 - h_3 = \frac{1}{2}(W_3^2 - W_2^2) = \frac{1}{2}(W_{3\theta}^2 + V_x^2 - W_{2\theta}^2 - V_x^2) = \frac{1}{2}(W_{3\theta} + W_{2\theta})(W_{3\theta} - W_{2\theta})$$

The axial velocity V_x is constant so $V_{1x} = V_{2x} = V_{3x} = V_x$. Recall the definition of Reaction R and substitute for $h_2 - h_3$ and $h_1 - h_3$ to give:

$$R = \frac{\Delta h_{rotor}}{\Delta h_{stage}} = \frac{h_2 - h_3}{h_1 - h_3} = \frac{\frac{1}{2}(W_{\theta 3} + W_{2\theta})(W_{3\theta} - W_{2\theta})}{U_m(W_{2\theta} - W_{3\theta})} = -\frac{1}{2}\frac{(W_{3\theta} - W_{2\theta})}{U_m}$$

We can write $W_{3\theta} = V_x \tan\beta_3$, and $W_{2\theta} = V_x \tan\beta_2$ from a generic velocity triangle so:

$$R = -\frac{1}{2}\frac{V_x}{U_m}(\tan\beta_2 + \tan\beta_3)$$

V_x/U_m is the flow coefficient so this expression can be simplified slightly:

$$R = -\frac{1}{2}\phi(\tan\beta_2 + \tan\beta_3) \tag{8.1}$$

It is very useful to change Equation 8.1 into one that uses α_2 instead of β_2 this form of the equation is particularly useful as it uses the stator geometry.

Apply the key rule in the tangential direction so:

$$W_{2\theta} = V_{2\theta} - U_m = V_x \tan\alpha_2 - U_m$$

Insert expressions for $W_{2\theta}$ and $W_{3\theta}$ into the previous equation for reaction:

$$R = -\frac{1}{2}\frac{(W_{3\theta} - W_{2\theta})}{U_m}$$

Substitute into Equation 8.1 to get:

$$R = -\frac{1}{2}\frac{V_x}{U_m}(\tan\alpha_2 + \tan\beta_3) + \frac{1}{2}\frac{U_m}{U_m}$$

which simplifies to:

$$R = \frac{1}{2} - \frac{\phi}{2}(\tan\alpha_2 + \tan\beta_3) \tag{8.2}$$

Example A turbine stage of a reaction 0.4 is to operate at a flow coefficient of 1.0, if the stator exit flow angle is 60°. Estimate the rotor relative inlet and exit flow angles.

Solution This example is quite easy but drives home the point that once the non-dimensional flow parameters are set the geometry of the machine is fixed.

The Stator exit flow angle is known α_2 along with the reaction, R and the flow coefficient ϕ Rearrange Equation 8.2:

$$\tan\beta_3 = \frac{1 - 2R}{\phi} - \tan\alpha_2$$

and then substitute the values we are given:

$$\beta_3 = \tan^{-1]}\left(\frac{1 - 2\times 0.4}{1.0} - \tan 60^\circ\right) = -56.9^\circ$$

We now examine two special cases of zero reaction (also called impulse blading) and 50% reaction to illustrate the fundamental trade-offs that are made when varying the level of reaction. In practise machines will operate with varying levels of reaction but comparing two very different cases gives information about the general trends.

8.1.1 Zero Reaction (Impulse) Stage

If $R = 0$, from Equation 8.2 it is apparent that $\tan\beta_2 = -\tan\beta_3$ So the inlet and outlet of rotor have equal and opposite and opposite angles. This leads to bucket shaped blades.

$$|W_3| = |W_2| \text{ and } \beta_3 = -\beta_2$$

A sketch of an impulse blade is shown in Figure 8.2 on the left hand side. We saw earlier that for a rotor row Equation 5.20 that energy conservation is equivalent to:

$$h_{0rel1} - \frac{U_1^2}{2} = h_{0rel2} - \frac{U_2^2}{2}$$

Since there are no changes in blade speed between the inlet and the outlet of the rotor then $h_{0rel1} = h_{0rel2}$ which means that for the zero reaction case:

$$h_{0rel1} = h_{0rel2} \implies h_1 + \frac{W_1^2}{2} = h_2 + \frac{W_2^2}{2} \implies h_1 = h_2$$

so there is no static enthalpy change across the rotor in an impulse blade. Since:

$$R \approx \frac{\Delta p_{rotor}}{\Delta p_{stage}}$$

there is no pressure change across rotor and all acceleration of the flow is across the stator, which is why it is often called a nozzle. Note that:

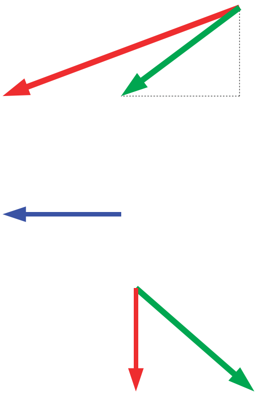
- Zero reaction has *high turning* in the rotor, typically $\beta_2 = 60^\circ = -\beta_3$ giving around 120° of flow turning in rotor.
- All the stagnation enthalpy drop must be across rotor, since only the rotor is moving. There is no work done in the stator.

Zero reaction is often called impulse blading as there is no pressure change in the rotor so the mechanism of turning the rotor is down to kinetic energy changes. Where $R > 0$ there is a pressure drop in the rotor passage so the mechanism of energy transfer is more complex. The Euler equation however Equation 5.12 shows that the detailed mechanisms within the blade row do not determine the work input and output from the stage - it is the change in angular momentum through the rotor passage that matters.

8.1.2 50% Reaction Stage

It is useful to examine what happens when the reaction level is 50%.

Equation 8.1:



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this is the work output from each *stage* of the machine. The number of stages is given by the machine enthalpy drop divided by the stage enthalpy drop or:

$$N_{stages} = \frac{891}{55.5} = 16.05$$

This would be taken as sixteen stages for reaction blading.

The determination of blade geometry for reaction blades is straightforward, a cascade view and velocity triangle are shown in Figure 8.2. We know that the rotor blades are the mirror image of the stator blades. Therefore $\beta_2 = -\alpha_3 = 0$ as there is no exit swirl. Stator and rotor blades are a mirror image: $\beta_3 = -\alpha_2 = -75°$. The deflection across the rotor is therefore $75°$. Compared to the impulse blading the 50% reaction blading gives lower turning and therefore lower work output, necessitating more stages for the same power output. The advantage of reaction blading is a slightly higher efficiency.

In practise the design is not limited to a single choice of reaction, so one might end up with four impulse stages in a high pressure turbine and eight stages of 50% reaction in a low pressure turbine.

Example In the previous example the turbine is designed to produce 100 MW of power. At inlet the steam density is around 3.26 kg/m^3 (this can be checked with steam tables). Estimate the local blade height h.

Solution The first step is to find the mass flow.

$$P = \dot{m}w \implies \dot{m} = \frac{P}{w} = \frac{100 \times 10^6}{891 \times 10^3} = 112.2\ kg/s$$

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From continuity (mass conservation):

$$\dot{m} = \rho V_x A = \rho V_x (2\pi r_m h) \implies h = \frac{\dot{m}}{\rho V_x 2\pi r_m}$$

The axial velocity $V_x = 126\, m/s$ and the steam density $\rho = 3.26\, kg/m^3$ so:

$$h = \frac{112.2}{3.26 \times 126 \times 2\pi \times 0.75} = 0.058\, m$$

The rationale behind these examples is to show that even with a relatively small amount of theory one can obtained powerful insights into the reasons behind the design choices made by turbomachinery designers.

8.7 Problems

1. A steam power plant has a High Pressure (HP) turbine, an Intermediate Pressure (IP) turbine and a Low Pressure (LP) turbine. All stages of the HP, IP and LP turbines are designed for an axial leaving velocity and a rotational speed of 3000 RPM. The axial velocity is kept a constant through each turbine.

   - HP turbine: stagnation pressure drop from $15 MN/m^2$ to $3 MN/m^2$; isentropic turbine (not stage) efficiency (total-to-total) of 0.88; impulse blading; stator exit flow angle of $75°$; blade mean radius of $0.51 m$.
   - IP turbine: stagnation pressure drop from $3 MN/m^2$ to $0.3 MN/m^2$; turbine isentropic efficiency of 0.88; 50% reaction blading; stator exit flow angle of 70°; blade mean radius of $0.74 m$.
   - LP turbine: stagnation pressure drop from $0.3 MN/m^2$ to $4 kN/m^2$; turbine isentropic efficiency of 0.84; 50% reaction blading; stator exit flow angle of 70°; blade mean radius of $1.09 m$.

   Superheated steam enters the HP turbine axially at a stagnation temperature of $550°C$. The steam is reheated to $550°C$ between the exit from the HP turbine and the inlet to the IP turbine.

   (a) Use the steam tables or chart to find relevant energy changes and estimate the number of stages required in the HP, IP and LP turbines (the kinetic energy leaving the LP turbine may be neglected).
   (b) Calculate the axial velocity for the HP, IP and LP turbines
   (c) Estimate the relative flow angles at exit from the rotor blades for each turbine

   Answers: 1) 8 stages, 11 stages, 5 stages, 2) $86\, m/s, 85\, m/s, 125\, m/s$, *3)* $-61.8°, -70°, -70°$

2. The repeating stage of an axial flow steam turbine is designed with the following flow angles: Exit flow from stator, $\alpha_2 = 75°$; relative flow at inlet to rotor, $\beta_2 = 45°$; relative flow at outlet from rotor, $\beta_3 = -68°$. Sketch the velocity triangles at the stator exit-rotor inlet and at the rotor outlet.

3. For the machine in the previous question Show that the values of flow coefficient and stage loading coefficient for the stage are $\phi = 0.366, \psi = 1.272$.

4. An intermediate pressure steam turbine is to be designed with 7 repeating stages as described above. The inlet conditions are: total temperature = $550°$, total pressure = $4.0\ MN/m^2$. The exit total pressure is $0.5\ MN/m^2$. The rotational speed of the turbine is to be $1500\ rev/min$ and the power output required is $90\ MW$. It is estimated that the turbine efficiency will be 90%. Calculate the mass flow through the turbine, the axial velocity, mean blade speed and the mean diameter of the turbine. *Answers:* $162\ kg/s$; $91.4\ m/s$; $249.7\ m/s$; $3.18\ m$

Chapter 9

Hydraulic Turbines

Hydro-electric power accounts for up to 20% of the world's electrical generation. Hydraulic turbines come in a variety of shapes determined by the available head and a number of sizes determined by the flow rate through the device. This chapter introduces no new concepts but instead allows the analysis techniques learnt over the preceding chapters to be applied by means of analysis examples.

Three three basic hydraulic machines (Pelton, Francis and Kaplan) are discussed and with all hydraulic machines the simplification for analysis and operation is that the fluid density is constant but the complication is that cavitation may occur in the machine. Cavitation is effectively the boiling of the liquid as it is exposed to extremely low pressures in certain parts of the turbine.

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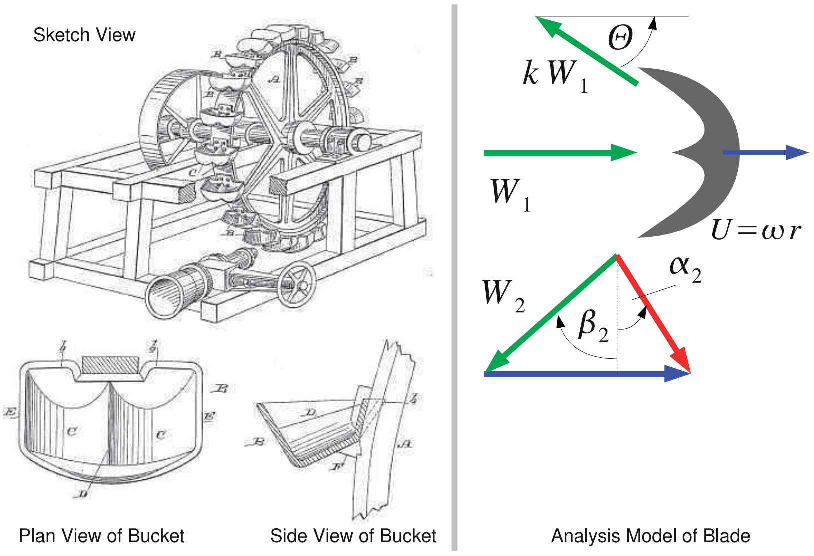

Sketch View

Plan View of Bucket

Side View of Bucket

Analysis Model of Blade

Figure 9.1: Pelton's Patent Application and Analysis Model

9.1 Pelton Wheel

This device invented around 1880 by Lester Pelton is used in high head locations with 300m to 4000m head, it has a peak efficiency of around 90%.

Figure 9.1 shows the Pelton wheel on the left and the sketch on the right shows a diagram of operation with velocity triangles.

The analysis of this machine is straightforward and can be found in any basic fluid mechanics textbook (e.g. Massey (1989)) as a simple control volume can be drawn around the rotor blade and the force determined from a simple analysis. Alternatively the results can be obtained from the Euler equation as follows.

Recall that $P = \dot{m}\omega(V_{2\theta}r_2 - V_{1\theta}r_1)$ so the key parameters to work out are the inlet tangential velocity and the exit tangential velocity.

At inlet the situation is straightforward $V_{1\theta} = V_j$ the jet velocity, as the nozzle directs the flow only in the tangential direction. At exit the velocity triangle shown in Figure 4.11 must be used.

Note that : $W_1 = V_j - U$ so $W_2 = kW_1 = k(V_j - U)$ where k is an empirical coefficient for frictional losses in the bucket. From the velocity triangle:

$$V_{2\theta} = U + W_{2\theta}$$

and that $W_{2\theta} = -W_2 \cos\Theta$, where Θ is the bucket angle shown in Figure 9.1. angle made by the bucket. The negative sign arises as our sign convention is "positive in the direction of rotation". Substituting yields:

$$V_{2\theta} = U - W_2\cos\Theta = U - k(V_j - U)\cos\Theta = -kv_j\cos\Theta + U(1 + k\cos\Theta)$$

Substitute this into the expression for power:

$$P = \dot{m}U(V_j - U)(1 + k\cos\Theta)$$

In the Pelton wheel all the pressure drop occurs in the stator or nozzle so the machine can be classified as an impulse machine.

9.2 Analysis Approach

The analysis of the Pelton wheel is extremely straightforward, other hydraulic machines required a more nuanced approach. This is a three step process:

1. Given the *flow rate* apply the *continuity* equation to get the radial or axial velocity.
2. The *geometry* (blade angles and radii) will yield absolute and relative velocities by means of velocity triangles.
3. *Power* is obtained from the Euler Equation.

Empirically obtained values of efficiency (or estimated values based on prior experience) link the power produced and the head drop across the machine. There are two methods of doing this depending of what information is available.

1. If the total head is given this yields the ideal power:

   $$P_{ideal} = \rho\dot{Q}gH = \dot{m}gH$$

   Given the ideal power, we have the actual power (from the Euler equation above) this yields the efficiency.

2. If loss information is given, the actual head is obtained from:

   $$H_{actual} = \frac{P_{actual}}{\rho\dot{Q}g} + H_{loss}$$

   The ideal head is given by $H_{actual} - H_{loss}$ so given the actual power and the losses efficiency can be obtained.

This may seem slightly abstract at the moment but will become more concrete as the examples are explained. Firstly a Francis turbine analysis is conducted where the overall head is given and the draft tube and supply pipe losses must be estimated, secondly an analysis on a Kaplan turbine is presented where losses are calculated for each component.

9.3 Francis Turbine

The Francis turbine was briefly described in Chapter 4. But for this analysis of the machine each station through the device is considered in more detail, a diagrammatic representation of the turbine is shown in Figure 9.2 with each of the analysis stations labelled. The four stations are:

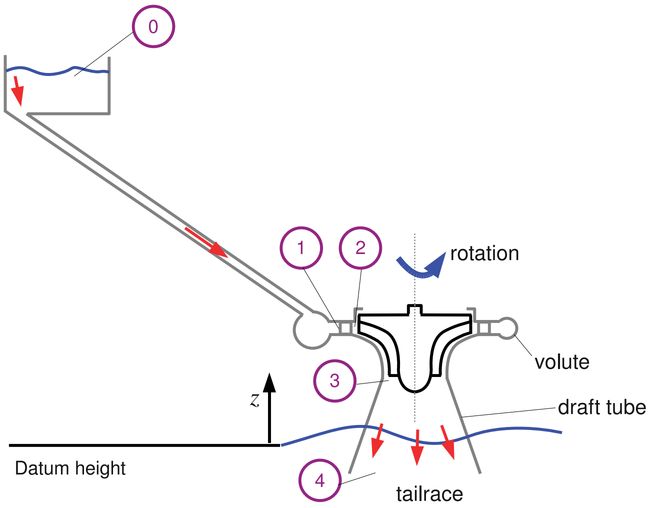

Figure 9.2: Analysis of a Francis Turbine

1. Inlet to Guide Vanes. (Station 1) There may be a small amount of swirl at inlet to the guide vanes because of the use of a volute to distribute the flow
2. Exit from Guide Vanes. (Station 2) Flow is deflected by the guide vanes (stator) to give high swirl velocity, and thus high angular momentum. Draw the velocity triangle to get the relative flow at inlet to the runner blades.
3. Exit from Runner. (Station 3) The aim here is to design at exit for zero absolute swirl as otherwise kinetic energy is wasted. Again we must draw the velocity triangle to get relative flow at exit from runner.
4. Draft Tube. (Station 4) This diffuser allows recovery of some of the exit kinetic energy of the axial flow.

Unlike the Pelton wheel all the pressure change does not occur over the stator so the Francis turbine can be thought of as a reaction machine. Flow from station to 1 station 2 through guide vanes is radially inwards, while being deflected and gaining angular momentum.

First we apply continuity: Flow rate $Q_1 = Q_2$:

$$Q_2 = V_{2r} 2\pi r_2 b_2 \text{ and } Q_1 = V_{1r} 2\pi r_1 b_1$$

If $b_1 = b_2$, (constant width vanes), then $V_{2r} r_2 = V_{1r} r_1$. Now the guide vanes have a non-zero thickness, and this introduces a blockage t at station 2 so to allow for this:

$$Q_2 = 2\pi r_2 b_2 (1 - t) V_{2r}$$

Typical $t = 0.08$ or about 8% of the area is blocked by the inlet guide vanes. The detailed analysis process is as follows:

- From the value of V_{2r}, the flow angle at exit from the vanes, α_2 and the blade speed, ωR_2, construct the velocity triangle at vane exit, 2

- Usually the design is for no exit swirl to reduce the exit kinetic energy that is lost, i.e. $V_{3\theta} = 0$. Thus from the blade speed ωR_3, the velocity triangle at runner exit, 3 can be constructed.

- The runner inlet and exit relative flow angles (to give the geometry of the runner blades) are obtained and also the power from the change in angular momentum across the runner (Euler equation).

- Finally to get the efficiency, the height of the reservoir and information on losses is required.

Example Consider a machine with the following specification:

- Outer diameter of runner, $2r_2 = 2.0\,m$

- Rotational speed, $N = 200\,rev/min$

- Guide vane height, $b_2 = 0.3\,m$

- Vane blockage, $t = 0.08$

- Vane exit angle, $\alpha_2 = 75^\circ$ from the radial direction

- Impeller designed for axial flow at exit, $\alpha_3 = 0$, and $r_{3m} = 0.5\,m$; $b_3 = 0.4\,m$

- Supply head, $H_O = 63\,m$; Flow rate $Q = 12\,m^3/s$

- Flow losses: $2\,m$ of heat loss in the supply pipe, $0.5\,m$ of head loss in the draft tube

- Draft tube velocity of $4\,m/s$

Determine the relative angles at inlet and exit of runner to give a preliminary design for the runner geometry. Also find the power output.

Solution The strategy is to apply the analysis steps outlined earlier. The most difficult part is drawing the velocity triangles at station 2 and station 3.

Continuity at 2 At exit from guide vanes:

$$Q = 2\pi r_2 b_2 (1-t) V_{2r} \implies 12 = 2\pi \times 1.0 \times 0.3(1-0.08) V_{r2}$$

Working the numbers though yields: $V_{2r} = 6.92\,m/s$

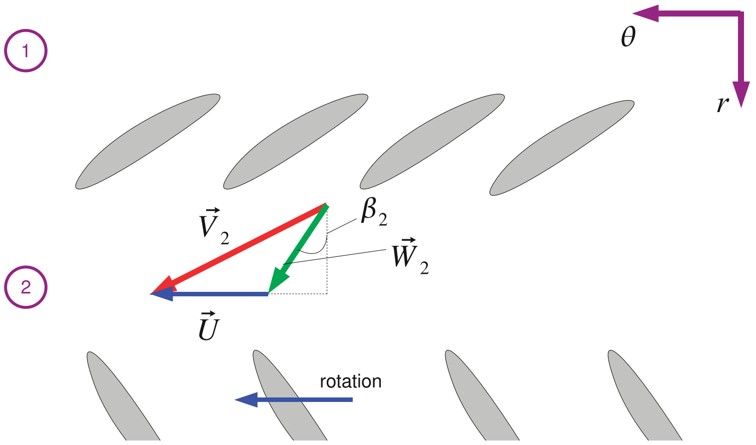

Figure 9.3: Velocity Triangle for Francis Turbine Guide Vane Exit

Velocity Triangle at 2: Figure 9.3 shows the velocity triangle at 2 and the corresponding cascade view. Earlier velocity triangles were drawn for pumps in a very similar manner. Since the geometry of the blades is very complex only the first part of the blade is drawn. To determine power the Euler equation only requires values at inlet and exit of the turbine so a sketch at inlet and exit will suffice. In reality the turbine blade curves around into the axial direction which can be represented using CAD for example but is not easy in a sketch. The velocity triangle is in the tangential radial plane.

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$$V_{2\theta} = V_{2r}\tan\alpha_2 = 6.92\tan 75^\circ \implies V_{2\theta} = 25.83\,m/s$$

$$V_2 = \sqrt{V_{2r}^2 + V_{2\theta}^2} = \sqrt{25.83^2 + 6.92^2} = 26.74\,m/s$$

From the velocity triangle:

$$V_{2\theta} = W_{2\theta} + \omega r_2$$
$$\omega r_2 = 2\pi \times \frac{200}{60} \times 1.0 = 20.94\,m/s$$
$$\Rightarrow W_{2\theta} = V_{2\theta} - \omega r_2 = 25.83 - 20.94 = 4.89\,m/s$$

So we can now work out the relative flow angle at 2:

$$\tan\beta_2 = \frac{W_{2\theta}}{V_{2r}} \implies \beta_2 = \tan^{-1}\left(\frac{4.89}{6.92}\right) = 35.2^\circ$$

In Chapter 4 the velocity of a jet expanding to atmospheric pressure was shown to be $\sqrt{2gH}$ for this turbine the ratio, $V_2/\sqrt{2gH} = 0.76$ so not all the acceleration of the fluid occurs over the stator so the reaction $R > 0$ and the machine is not an impulse turbine.

Continuity at 3: The flow area is given by $2\pi r_{3m} b_3$ so:

$$Q = 2\pi r_{3m} b_3 V_{3x}$$

Rearrange and put the numbers in to give V_{3x}:

$$V_{3x} = 12/\left(2\pi \times 0.5 \times 0.4\right) = 9.55\,m/s$$

Velocity Triangle at 3: Figure 9.4 shows the velocity triangle at 3 and the corresponding cascade view, again the complex geometry in the radial direction is not drawn instead only the exit of the runner is considered.

No exit swirl, $V_{3\theta} = 0$

$$\therefore W_{3\theta} = -\omega r_{3m} = -2\pi \times \frac{200}{60} \times 0.5 = -10.47\,m/s$$

So we can now work out the relative flow angle:

$$\tan\beta_3 = \frac{W_{3\theta}}{V_{3x}} \implies \beta_3 = \tan^{-1}\left(\frac{-10.47}{9.549}\right) = -47.6^\circ$$

Power output: Recall that:

$$P = \dot{m}\omega\left(r_2 V_{2\theta} - r_{3m} V_{\theta 3}\right)$$

but $V_{3\theta} = 0$ so:

$$P = \dot{m}\omega r_2 V_{2\theta} = 1000 \times 12 \times 2\pi \times \frac{200}{60} \times 25.82 = 6.49\,MW$$

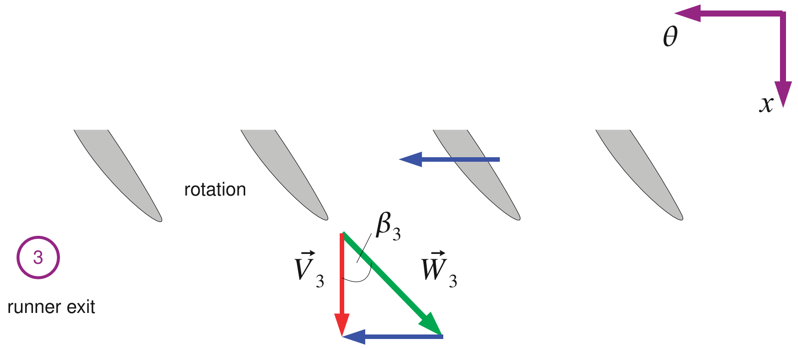

Figure 9.4: Velocity Triangle for Francis Runner Exit

Draft Tube Analysis Although external to the rotating parts of the turbine the draft tube is an important part of the hydraulic machine. The draft tube is a conical diffuser with around 7° divergence which reduces the exit kinetic energy in the departing fluid and therefore increases the efficiency of the machine as a whole.

In this example the aim is to reduce exit kinetic energy at station 4, note that h_4 is normally below the height of the river that the flow exits into.

As seen in Chapter 4 the total head at station 0 (the reservoir) is:

$$\frac{p_o}{\rho g} + \frac{V_o^2}{2g} + z_o$$

But $p_o = 0$ as in hydraulics we always use gauge pressure where zero is the ambient atmospheric pressure, in the reservoir $V_0 = 0$ so the total head at station zero is simply given by $H_0 = z_0$

We need to find the head across turbine:

$$\Delta H = H_1 - H_3$$

Now $H_1 = H_o - h_{fp}$, where h_{fp} is the pipe head loss due to friction. Now examine the total head from the other end of the machine starting at station 4. If loss in draft tube is given by h_{fDT} then:

$$H_4 = H_3 - h_{fDT} \implies H_3 = H_4 + h_{fDT}$$

The total head at station 4 is given by:

$$H_4 = \frac{p_4}{\rho g} + \frac{V_4^2}{2g} + z_4$$

The pressure at station 4 must be equal to the pressure of the fluid in the river as a whole so is given by the pressure due to a height of fluid: $p_4 = \rho g h_4$. Also $z_4 = -h_4$ substituting gives:

$$H_4 = \frac{\rho g h_4}{\rho g} + \frac{V_4^2}{2g} - h_4 = \frac{V_4^2}{2g}$$

Thus

$$H_3 = h_{fDT} + \frac{V_4^2}{2g}$$

So head across turbine:

$$H_1 - H_3 = H_o - h_{fp} - \left(h_{fDT} + \frac{V_4^2}{2g}\right)$$

simplifying slightly:

$$H_1 - H_3 = H_o - \frac{V_4^2}{2g} - h_{fp} - h_{fDT} \tag{9.1}$$

So for maximum head change across turbine and therefore the maximum power the design aim is to keep V_4, h_{fp} and h_{fDT} as low as possible.

To apply this information on pipe flow losses and draft tube losses is required and the velocity at station 4. In this example the information is provided: $h_{fp} = 2\,m, h_{fDT} = 0.5\,m, V_4 = 4\,m/s$.

h_{fp} can be calculated from a simple knowledge of pipe flow friction (Massey (1989)). Draft tube losses require some empirical knowledge and V_4 is set by conservation of mass. In this example $V_3 = V_{3x} = 9.55\,m/s$, so the exit velocity is considerably reduced by the draft tube.

Apply Equation 9.1 with the above numbers:

$$H_1 - H_3 = 63 - \frac{4^2}{2g} - 2 - 0.5 = 59.68\,m$$

The turbine efficiency may then be worked out:

$$\eta_T = \frac{P_{actual}}{P_{ideal}} = \frac{6.49 \times 10^6}{1000 \times 12 \times 59.68 \times 9.81} = 0.92$$

9.4 Kaplan Turbine

The Kaplan turbine is shown in diagrammatic form in Figure 9.5 which also shows the analysis stations. In comparison to the analysis of the Francis turbine an extra analysis station is needed (station 3) between the guide vane exit and the runner inlet as the flow is turned through 90° before entering the turbine blades. The analysis approach is as follows:

1. From station 2, the guide vane exit, to station 3, the runner inlet one can either use conservation of angular momentum if guide vanes are radial. In some Kaplan turbines the guide vanes are orientated in the axial direction in which case station 2 = station 3. The velocity triangles are then drawn to get relative velocities at rotor inlet.

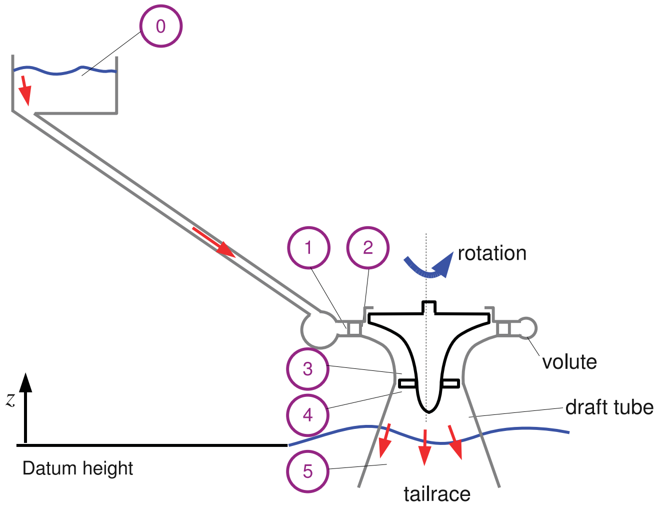

Figure 9.5: Analysis of a Kaplan Turbine

2. At 4, runner exit, draw the velocity triangle to obtain relative and absolute exit velocities.

3. The Euler equation 5.12 gives the power $P = \dot{m}\omega \left(r_3 V_{3\theta 3} - r_4 V_{4\theta} \right)$

4. Calculate losses. In this example component losses will be considered. Component head losses will be obtained by working through the stations from 0 to 4. This gives the turbine efficiency.

5. Finally the draft tube, station 4 to station 5 is analysed, this calculation will give us the overall efficiency of the site.

Example Consider a Kaplan Turbine with the following characteristics:

- Guide Vane Exit: Radial flow, $r_2 = 2.0\ m$; blade height, $b_2 = 1.0\ m$, blockage, $t_2 = 0.08$; absolute flow angle to radial, $\alpha_2 = 50°$
- Runner: Mean radii at station 3 and 4 are the same: $r_{3m} = r_{4m} = 0.85\ m$, blade height, $b_3 = 0.7\ m$; designed for zero exit absolute swirl, $V_{4\theta} = 0$; rotational speed, $N = 300\ rev/min$.
- Draft Tube: Reduces axial velocity to $V_5 = 5\ m/s$
- Flow rate: $Q = 36\ m^3/s$

Calculate the runner relative and absolute flow angles and velocities at inlet and exit. Also calculate the power output.

Solution The strategy is to apply the analysis steps outlined earlier. In a Kaplan turbine the flow through the guide vanes may be be radial or axial so the geometry must be carefully inspected.

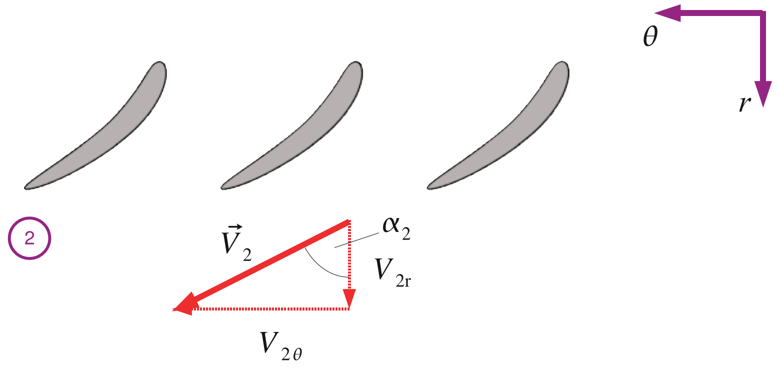

Figure 9.6: Velocity Triangle for a Kaplan Turbine at Guide Vane Exit

Guide Vane Exit to Runner Inlet At guide vane exit (station 2), the flow is radial. Apply continuity: $Q = 2\pi r_2 b_2(1 - t_2)V_{2r}$, where V_{2r} is the *radial* velocity at 2 so:

$$V_{2r} = \frac{36}{2\pi \times 2.0 \times 1.0 \times (1 - 0.08)} = 3.11\, m/s$$

From the velocity triangle at 2 (Figure 9.6):

$$V_{2\theta} = V_{2r} \tan\alpha_2 = 3.11 \times \tan 50^\circ = 3.71\, m/s$$

$$V_2 = \sqrt{(V_{2r})^2 + (V_{2\theta})^2} = \sqrt{3.11^2 + 3.71^2} = 4.84\, m/s$$

Since there are no blades between station 2 and station 3, angular momentum is conserved, so:

$$r_{3m}V_{3\theta} = r_2 V_{2\theta} \implies V_{3\theta} = \frac{r_2 V_{2\theta}}{r_{3m}} = \frac{2.0 \times 3.71}{0.85} = 8.73\, m/s$$

At station 3 the flow is axial. The continuity equation is: $Q = 2\pi r_{3m} b_3 V_{3x}$, where V_{3x} is the *axial* velocity at 3.

$$V_{3x} = \frac{36}{2\pi \times 0.85 \times 0.7} = 9.63\, m/s$$

From the velocity triangle at 3 (Figure 9.7):

$$V_3 = \sqrt{(V_{3x})^2 + (V_{3\theta})^2} = \sqrt{9.63^2 + 8.73^2} = 13.00 m/s$$

$$\alpha_3 = \tan^{-1}\left(\frac{V_{3\theta}}{V_{3x}}\right) = \tan^{-1}\left(\frac{8.73}{9.63}\right) = 42.2^\circ$$

The blade blade speed U_3 is given by:

$$\omega r_{3m} = \frac{2\pi N}{60} r_{3m} = \frac{2\pi \times 300}{60} 0.85 = 26.70\, m/s$$

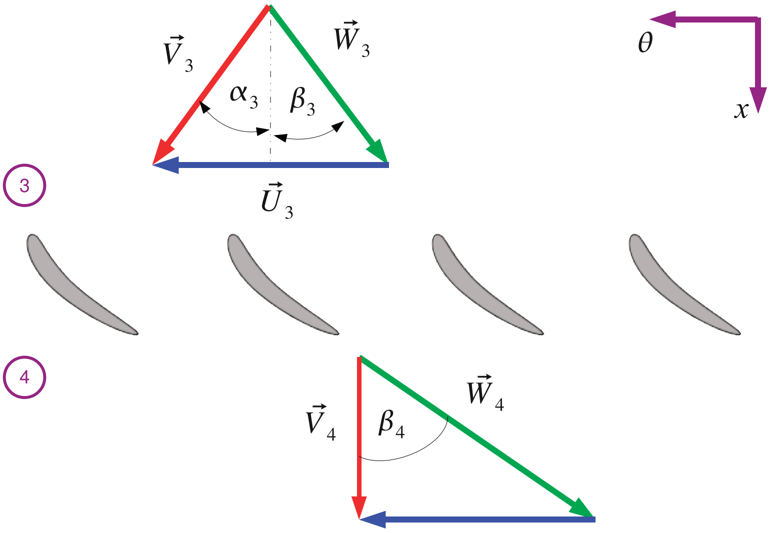

Figure 9.7: Velocity Triangle for a Kaplan Runner

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From the velocity triangles (or from the key rule (Equation 2.1) in the tangential direction):

$$V_{3\theta} = U + W_{3\theta} \implies W_{3\theta} = V_{3\theta} - \omega r_{3m} = 8.73 - 26.70 = -17.97\, m/s$$

we now have all the ingredients to work out the relative flow angle at inlet to the runner:

$$\beta_3 = \tan^{-1}\left(\frac{W_{3\theta}}{V_{3x}}\right) = \tan^{-1}\left(\frac{-17.97}{9.63}\right) = -61.8^\circ$$

Across Runner At runner exit, $V_{4\theta} = 0$. $V_{4x} = V_{3x}$ as $r_{4m} = r_{3m}$, hence from the velocity triangle at 4:

$$W_{4\theta} = -\omega r_{4m} = -\omega r_{3m} = -27.60\, m/s$$

$$\beta_4 = \tan^{-1}\left(\frac{W_{4\theta}}{V_{4x}}\right) = \tan^{-1}\left(\frac{-27.60}{9.63}\right) = -70.2^\circ$$

$$W_4 = \sqrt{W_{4x}^2 + W_{4\theta}^2} = \sqrt{17.97^2 + 27.60^2} = 28.38 m/s$$

Euler Equation

$$P = \dot{m}\omega\left(r_3 V_{3\theta 3} - r_4 V_{4\theta}\right)$$

But $V_{4\theta} = 0$ and the fluid is water with density, $\rho = 1000 kg/m^3$

$$P = 36 \times 1000 \times 26.70 \times 8.73 = 8391 kW$$

This is the power produced by the turbine.

9.4.1 Loss Estimation

One way losses may be calculated is to assume that across any component the total head loss is proportional to the velocity head at exit:

$$\Delta H = k\left(\frac{V^2}{2g}\right)_{exit}$$

This is a concept borrowed from minor losses in pipe flow calculations, this is still an empirical method as one has to specify an appropriate value of k for each component. For runners the relative dynamic head $W^2/2g$ is used as the and the cause of most fluid friction is the the relative motion between the runner surfaces and the mainstream flow.

Example With the Kaplan turbine from the previous example the loss coefficients may be taken as $k = 0.05$ for the guide vanes and bend and $k = 0.06$ for the runner. Ignoring pipe friction estimate the turbine efficiency.

Solution Each station is considered in turn and an appropriate value of k specified.

- $0 \Rightarrow 1$, for this example ignore the pipe loss. Though it is relatively easy to calculate.
- $1 \Rightarrow 2$, for the guide vanes, $k = 0.05$: $\Delta H = 0.05(V_2^2)/2g = 0.05(4.84^2)/2g = 0.06\,m$
- $2 \Rightarrow 3$, $k = 0.05$ for the bend from radial to axial flow: $\Delta H = 0.05(V_3^2)/2g = 0.05(13.0^2)/2g = 0.43\,m$
- $3 \Rightarrow 4$, for the runner use the relative exit head. With k = 0.06: $\Delta H = 0.06(W_4^2)/2g = 0.06(28.38^2)/2g = 2.46\,m$

From station 1 to station 4, across the whole turbine, the ideal total head drop for the actual power is given by:

$$\dot{m}g\Delta H_{ideal} = P_{actual} \implies \Delta H_{ideal} = \frac{P_{actual}}{\dot{m}g\Delta H_{ideal}}$$

this means that:

$$\Delta H_{ideal} = \frac{8391 \times 10^3}{36 \times 1000 \times 9.81} = 23.76\,m$$

which is the total head required if there were no losses in the turbine. The actual total head drop across the turbine,

$$H_1 - H_4 = \Delta H_{ideal} + \sum \Delta H_{losses}$$

Each of the losses have already been calculated so substituting:

$$H1 - H4 = 23.76 + (0.06 + 0.43 + 2.46) = 26.71\,m$$

This is the required total head drop across the turbine alone to produce the desired flow rate. The turbine efficiency is:

$$\eta_T = \frac{\text{Actual Power}}{\dot{m}g\,(H_1 - H_4)} = \frac{8391 \times 10^3}{36 \times 1000 \times 9.81 \times 26.71} = 0.89$$

This is the efficiency for the turbine alone.

9.4.2 Draft Tube Analysis

Normally in hydraulic machines two efficiencies may be quoted. The first known as “turbine efficiency” excludes draft tube and inlet pipe losses from the definition of available head and gives an indication as to the “goodness” of the turbine runner. The second known as “system efficiency” includes the losses in inlet pipes and draft tube and is of crucial interest to those designing or operating particular hydro-electric power plants.

Example Using the Kaplan turbine in the previous example. Estimate the system efficiency if the draft tube loss coefficient is given by $k = 0.2$.

Solution From station 4 to 5 the draft tube reduces the axial velocity and hence the wasted exit kinetic energy. The total head at 5 is given by:

$$H_5 = \frac{p_5}{\rho g} + \frac{V_5^2}{2g} + z_5$$

But $z_5 = -h_5$ and so $p_5 = \rho g h_5$ as the pressure at station 5 must be equal to the pressure in the river so:

$$H_5 = \frac{p_5}{\rho g} + \frac{V_5^2}{2g} + z_5 = \frac{V_5^2}{2g} = \frac{5^2}{2 \times 9.81} = 1.27\,m$$

Now $\Delta H_{4-5} = H_4 - H_5$, so $H_4 = H_5 + \Delta H_{4-5}$ and $k = 0.2$ so the draft tube loss becomes:

$$\Delta H_{4-5} = k\left(\frac{V_4^2 - V_5^2}{2g}\right) = 0.2 \times \left(\frac{9.63^2 - 5^2}{2 \times 9.81}\right) = 0.69\,m$$

This gives $H_4 = 1.27 + 0.69 = 1.96\,m$ so the inlet head to the turbine $H_1 = (H_1 - H_4) + H_4 = 26.71 + 1.96 = 28.67\,m$

This is the inlet total head required for the turbine to deliver the actual power. So the overall overall efficiency for the system is given by:

$$\eta_S = \frac{\text{Actual Power}}{\dot{m}gH} = \frac{8391 \times 10^3}{36 \times 1000 \times 9.81 \times 28.67} = 0.83$$

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9.4.3 Effect of Draft Tube

To make clearer the influence of the draft tube, a calculation is carried out to illustrate what would happen if it was replaced with a parallel pipe, $V_5 = V_4 = 9.63\ m/s$ and assuming that there is no head loss associated with the parallel pipe. Consider what happens to the head and efficiency when there is a parallel pipe.

$$H_4 = H_5 = \frac{V_5^2}{2g} = \frac{9.63^2}{2 \times 9.81} = 4.73m$$

This makes the inlet head to the turbine $H_1 = (H_1 - H_4) + H_4 = 26.71 + 4.73 = 31.44\, m$. For the same power output the inlet head would need to be raised from from $28.67\ m$ to $31.43\ m$. The changed overall efficiency for the turbine is given by:

$$\eta_S = \frac{\text{Actual Power}}{\dot{m}gH} = \frac{8391 \times 10^3}{36 \times 1000 \times 9.81 \times 31.44} = 0.76$$

So there is a substantial reduction in efficiency, the difference in cost between a parallel pipe and a diffuser is negligible so a draft tube is well worth having!

There is a drawback to having a draft tube, consider the turbine exit pressure for both the parallel pipe and the draft tube. The total head at station 4 is given by:

$$H_4 = \frac{p_4}{\rho g} + \frac{V_4^2}{2g} + z_4 = h_4 + \frac{V_4^2}{2g} + z_4$$

Where h_4 is the static head at station 4:

$$h_4 = H_4 - \frac{V_4^2}{2g} - z_4$$

For the sake of example say that the turbine exit is $1\ m$ above the tail race, that is $z_4 = 1.0\ m$ With the draft tube installed. $H_4 = 1.96\ m$ and $V_4 = 9.63\ m/s$ so the static pressure head at station 4 is:

$$h_4 = 1.96 - \frac{9.63^2}{2 \times 9.81} - 1.0 = -3.77\, m$$

The negative sign indicates that the pressure is below atmospheric pressure. Expressing this pressure in Pa rather than m of fluid yields:

$$p_4 = h_4 \rho g = -3.77 \times 1000 \times 9.81 = -36.9kPa \approx -0.4\, bar$$

Without the draft tube so with a cylindrical pipe at exit. $H_4 = 4.72\ m$ and $V_4 = 9.63\ m/s$ so the static pressure head at station 4 is:

$$h_4 = 4.72 - \frac{9.63^2}{2 \times 9.81} - 1.0 = -1.00\, m$$

Again this is below atmospheric pressure, converting to pressure rather than head gives:

$$p_4 = h_4 \rho g = -1.00 \times 1000 \times 9.81 = -9.8 kPa \approx -0.1\, bar$$

which is considerably greater.

The problem with excessively low pressures is that they can lead to a phenomenon known as *cavitation*. Cavitation occurs if the absolute pressure falls below the saturated vapour pressure, p_v, for the water temperature. Or in other words when the pressure drops too low the water starts to boil even at the ambient temperature encountered by hydraulic machines. Essentially bubbles of vapour will be formed. As the pressure rises again the bubbles collapse suddenly and very high instantaneous pressures can be created ($p > 500 bar$) which can cause significant erosion of blade surfaces and lower machine performance.

In practise dissolved air starts to come out of solution of the fluid first and so some margin between the saturated vapour pressure and the pressure the turbine operates at is required. This is normally done by using an empirical parameter known as Thoma's parameter:

$$\sigma = \frac{p - p_v}{\Delta H} \tag{9.2}$$

where p is the pressure of the fluid, p_v is the saturated vapour pressure and ΔH is the head drop across the machine. Charts of Thoma's parameter derived from practical experience are available. The problem of cavitation can be alleviated somewhat by reducing z_4, that is lowering the turbine closer to the river level or even below it.

9.5 Problems

1. A Francis turbine operates with the following data: Head: $100m$; Flow rate 36 m^3/s; Speed: 410 rev/min; Outer radius of runner 0.6 m; Absolute flow angle: 72°; Velocity: 36 m/s; Exit swirl angle: 0°

   Calculate the effective runner width (i.e. ignoring blockage) and relative flow angle at inlet, the power output and efficiency. *Answers:* $0.859\, m, 37.3°, 31.8\, MW$*,90.0%*

2. A water turbine develops 480 kW with a head of 12.2 m with an overall efficiency of 86% when fitted with a cylindrical draft tube of $1.5\, m$ diameter. If a conical draft tube of $2.25\, m$ exit diameter is substituted, calculate the increase in efficiency and power. Assume the flow rate, head, speed and losses in the machine are unaltered, and that there is no exit swirl. *Answers: 88.3%,* 493 kW

3. A vertical axis Kaplan Turbine is to be designed for an overall head of $9\, m$, delivering $800\, kW$ at a rotational speed of 150 rev/min. The vanes and runner are to be mounted in an annular duct of outer diameter 2 m and inner diameter 0.8 m. There is to be no exit swirl, and a draft tube reduces the axial velocity by a factor of two. Making a reasonable assumption for the overall efficiency, estimate the required flow rate and hence the flow angle at exit from the vanes and the relative flow angles at inlet and exit for the runner. The losses of total head are estimated as:

   (a) across the guide vanes, 6% of exit velocity head

(b) across the runner, 8% of the exit relative velocity head

(c) in the draft tube, 20% of the change in the velocity head.

Calculate the overall efficiency (for comparison with your estimate). *Answers:* $10.3\ m^3/s$, $60.5°$, $-46.5°$, $-70.5°$, *88.3%*

Chapter 10

Analysis of Pumps

This bulk of this chapter builds on the introduction to pumps in Chapter 4 by providing an example of pump analysis. There are only a few new concepts in this chapter instead the techniques of drawing velocity triangles, forming the cascade view and applying the basic equations are applied to pumps. Before the main example the Euler equation is applied to predict pump performance, pump diffuser performance discussed and pump loss coefficients are detailed.

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10.0.1 Pump Geometry and Performance

A general arrangement diagram of a pump is in Figure 4.1 on page 44. If station 1 is inlet to rotor and station 2 the outlet of the rotor, then the Euler equation gives:

$$P = \dot{m}\omega(r_2V_{2\theta} - r_1V_{1\theta})$$

Consider the efficiency of a pump:

$$\eta_p = \frac{IdealPower}{ActualPower} = \frac{\dot{m}gH}{\dot{m}\omega(r_2V_{2\theta} - r_1V_{1\theta})}$$

This means that the Head H developed over the turbine is:

$$H = \frac{\eta_p}{g}\omega(r_2V_{2\theta} - r_1V_{1\theta}) \tag{10.1}$$

So for a large head rise one can make $r_2 > r_1$ as well as relying on the change in tangential velocity V_θ. Recall that the key rule (Equation 2.1) can also be applied in the tangential direction only:

$$V_{2\theta} = W_{2\theta} + \omega r_2 = V_{2r}\tan\beta_2 + \omega r_2$$

The relationship $W_{2\theta} = V_{2r}\tan\beta_2$ can be seen in Figure 4.5 or worked out from a new velocity triangle. The velocity V_{2r} is obtained by applying the continuity equation at station 2:

$$Q = 2\pi r_2 b_2(1-t_2)V_{2r} \implies V_{2r} = \frac{Q}{2\pi r_2 b_2(1-t_2)}$$

Substitute this into the expression for $V_{2\theta}$ obtained earlier to give:

$$V_{2\theta} = \frac{Q\tan\beta_2}{2\pi r_2 b_2(1-t_2)} + \omega r_2$$

If there is no swirl at entry then $V_{1\theta} = 0$. (A reasonable assumption as the inlet is usually the flow from a pipe and pipe flow generally has no rotational motion). Substitute expressions for $V_{1\theta}$ and $V_{2\theta}$ into Equation 10.1 to give:

$$H = \frac{\eta_p}{g}\left(\left[\frac{\omega\tan\beta_2}{2\pi b_2(1-t_2)}\right]Q + \omega^2 r_2^2\right) \tag{10.2}$$

The pump power is obtained after a short manipulation:

$$P = \rho\left[\frac{\omega\tan\beta_2}{2\pi b_2(1-t_2)}Q^2 + \omega^2 r_2^2 Q\right] \tag{10.3}$$

Equations 10.2 and 10.3 provide powerful insights into the choices designers have in determining pump geometry and operation. Consider Equation 10.2. The pump head (or pressure) rise produced increases with increasing efficiency. At zero flow rate the head produced by the pump is determined by the rotational speed, radius and efficiency only, because Equation 10.2 with $Q = 0$ becomes:

$$H = \frac{\eta_p}{g}\left(\omega^2 r_2^2\right)$$

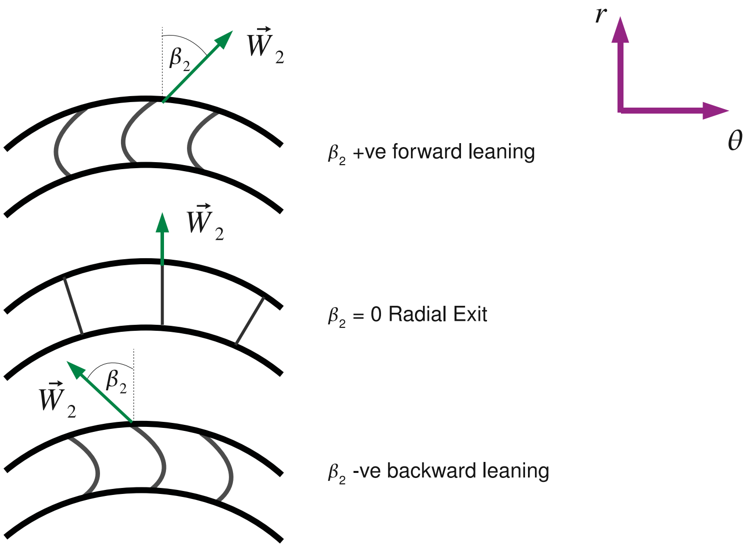
β_2
$\vec{W}_2$
r
θ
β_2 +ve forward leaning
$\vec{W}_2$
β_2 = 0 Radial Exit
$\vec{W}_2$
β_2
β_2 -ve backward leaning

Figure 10.1: Three Blade Angles at Impeller Exit

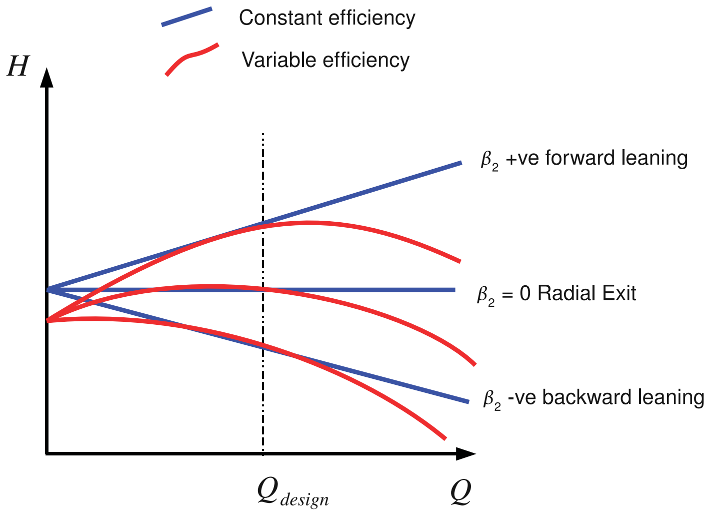

Figure 10.2: H vs Q for Three Blade Angles

The $\tan\beta_2$ term is also important. Consider three cases of radial pump design: firstly where β_2 is positive, this is often described as *forward leaning*, secondly where β_2 is zero called *radial exit* and finally β_2 is negative known as *backward leaning*. These three geometries are shown in Figure 10.1.

Equation 10.2 can be used to predict the performance of the pump as the flow rate varies at constant rotational speed. (This is generally achieved by adjusting valves at inlet and exit to the pump, though exactly how this is achieved is not important for this discussion).

The three cases are sketched on Figure 10.2 which shows two lines for each of the three pump geometries. The first of which is with constant efficiency (blue lines) so are straight lines on the graph as from Equation 10.2 H is linear with Q when everything else is constant. The three cases are:

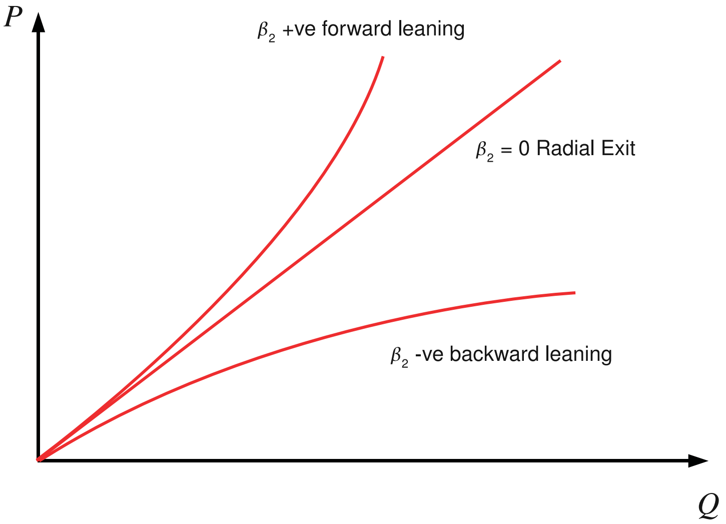

Figure 10.3: P vs Q for Three Blade Angles

- For $\beta_2 = 0$ (radial exit) then the head produced does not vary with flow rate, from Equation 10.2: $\tan\beta_2 = \tan 0 = 0$.
- For $\beta_2 > 0$ (forward leaning) the head produced increases with increasing flow rate, as $\tan\beta_2 > 0$.
- For $\beta_2 < 0$ (backward leaning) the head produced decreases with increasing flow rate.

The plots are then repeated with an efficiency that varies with the maximum value being at a design flow rate Q_{design} and reducing efficiency either side of the design value. These are the red lines shown in Figure 10.2 and show a more realistic characteristic curve for the pump. The key point is that with a simple equation the performance of pump as a whole has been characterised.

The power variation with flow rate can also be sketched using Equation 10.3 which is shown in Figure 10.3. The Q^2 term in equation 10.3 is significant. When $\beta_2 > 0$ this term results in the gradient of the P vs Q line increasing with increasing flow rate. For radial exit $\tan\beta_2 = 0$ so the gradient is constant and for backwards leaning $\tan\beta_2 < 0$ so the gradient reduces with increasing flow rate.

As a result of this backward leaning is generally preferred to forward leaning, despite the lower head produced. Backward leaning pumps have a higher efficiency due to lower exit swirl and forward leaning blades have an increasingly steep power curve. If Q is underestimated, the power requirement will be greatly underestimated so backward leaning blades are more tolerant of flow rate errors.

10.1 Pump Diffuser Analysis

For a machine with radial exit flow from the rotor, the kinetic energy may be reduced either by means of a vaned diffuser, or a vaneless diffuser. For a *vaneless* diffuser, as the flow goes out radially by continuity:

$$V_r \times r = \text{ constant } = K_1 \text{ or } V_r = K_1/r$$

By conservation of angular momentum:

$$V_\theta \, r = \text{ constant } = K_2 \text{ or } V_\theta = K_2/r$$

The exit kinetic energy is therefore given by:

$$\frac{V^2}{2} = \frac{V_\theta^2 + V_r^2}{2} = \frac{1}{2r^2}(K_1^2 + K_2^2)$$

So the exit kinetic energy reduces with the inverse square of the radius:

$$\frac{V^2}{2} \propto \frac{1}{r^2}$$

In a flow where the total pressure is conserved there will be a a corresponding increase in static pressure. If this pressure rise is large enough to cause separation then vanes may be required to control the flow.

10.2 Pump Losses

Losses are often expressed in terms of fractions of a dynamic head, in a similar manner to the technique used in the Kaplan turbine in Chapter 9. For an impeller the relative dynamic head at inlet

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is used. This is because fluid friction causes most of the loss and the relative motion between the impeller surface and the mainstream flow is the cause of much of that fluid friction. The head loss for an impeller is given as:

$$\Delta H_{Imp} = K_{Imp} \frac{W_1^2}{2g}$$

K_{Imp} is the impeller loss coefficient. For the diffuser the loss coefficient is based on the absolute exit velocity:

$$\Delta H_{Diff} = K_{Diff} \frac{V_2^2}{2g}$$

In actual fact the diffuser is the major source of loss in the pump. The loss in a diffuser may also be expressed in terms of a diffuser efficiency:

$$\eta_{Diff} = \frac{\Delta p_{actual}}{\Delta p_{ideal}} \tag{10.4}$$

where Δp_{ideal} is the ideal pressure rise with diffusion to zero velocity, so Δp_{ideal} is effectively the dynamic pressure at impeller exit or:

$$\Delta p_{ideal} = \frac{1}{2}\rho V_2^2 \text{ and } \eta_{Diff} = \frac{\Delta p_{actual}}{\frac{1}{2}\rho V_2^2} = \frac{p_3 - p_2}{\frac{1}{2}\rho V_2^2}$$

where station 3 is the diffuser exit. The diffuser head loss is given by:

$$\Delta H_{Diff} = H_2 - H_3$$

Total head is made up of pressure, velocity and height terms, assuming that the height terms are negligible the diffuser head loss can be rewritten:

$$\Delta H_{Diff} = H_2 - H_3 = \frac{p_2}{\rho g} + \frac{V_2}{2g} - \frac{p_3}{\rho g} - \frac{V_3}{2g} = \frac{p_2 - p_3}{\rho g} + \frac{V_2^2 - V_3^2}{2g}$$

In the diffuser V_3 will be very small so assuming $V_3 \approx 0$:

$$\Delta H_{Diff} = \frac{p_2 - p_3}{\rho g} + \frac{V_2^2}{2g} \implies p_3 - p_2 = \rho g \left(\frac{V_2^2}{2g} - \Delta H_{Diff} \right)$$

$$\implies \frac{p_3 - p_2}{\frac{1}{2}\rho V_2^2} = \frac{\rho g}{\frac{1}{2}\rho V_2^2} \left(\frac{V_2^2}{2g} - \Delta H_{Diff} \right) = 1 - \frac{\Delta H_{Diff}}{V_2^2/2g}$$

which can be written much more simply as:

$$\eta_{Diff} = 1 - K_{Diff} \tag{10.5}$$

which is useful if diffuser performance is expressed as an efficiency and a coefficient is needed or the other way around.

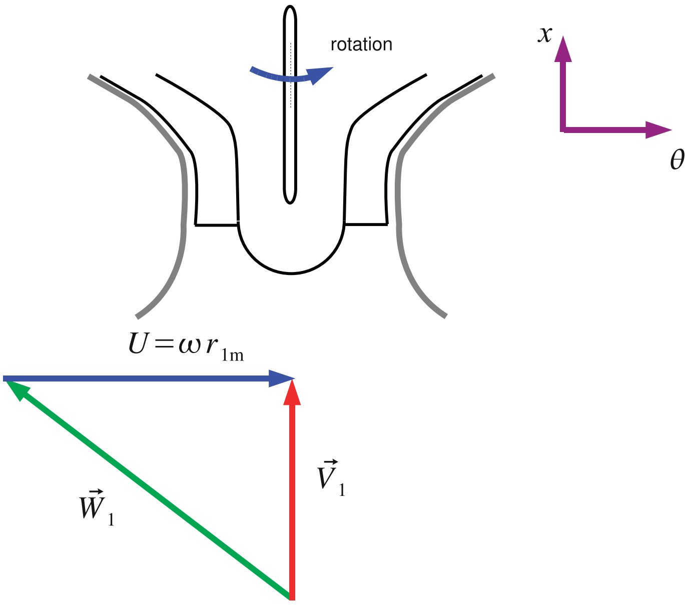


Figure 10.4: Inlet to Pump Impeller

10.3 Centrifugal Pump Example

Example Consider a centrifugal pump with data:

- At impeller inlet, mean radius, $r_{1m} = 0.1\,m$, blade height, $b_1 = 0.1\,m$
- At impeller exit, radius, $r_2 = 0.2\,m$, blade height, $b_2 = 0.03\,m$, blade blockage, t = 0.08
- Backward leaning blades with $\beta_2 = -20°$
- Flow rate of water, $Q = 0.2\,m^3/s$, Rotational speed, $N = 1450\,rev/min$
- Assume for losses: Impeller, $K_{imp} = 10\%$; Diffuser efficiency = 60%

Calculate: The relative flow at impeller inlet, absolute flow at impeller exit, the power required, head produced and the pump efficiency.

Solution The basic strategy is to work form pump inlet to exit drawing the cascade view and velocity triangles through the machine.

Impeller Inlet First sketch pump and velocity triangle at inlet (See Figure 10.4).

Continuity:

$$Q = 2\pi r_{1m} b_1 V_{x1} \implies V_{x1} = \frac{Q}{2\pi r_{1m} b_1} = \frac{0.2}{2\pi \times 0.1 \times 0.1} = 3.183\,m/s$$

Angular velocity:

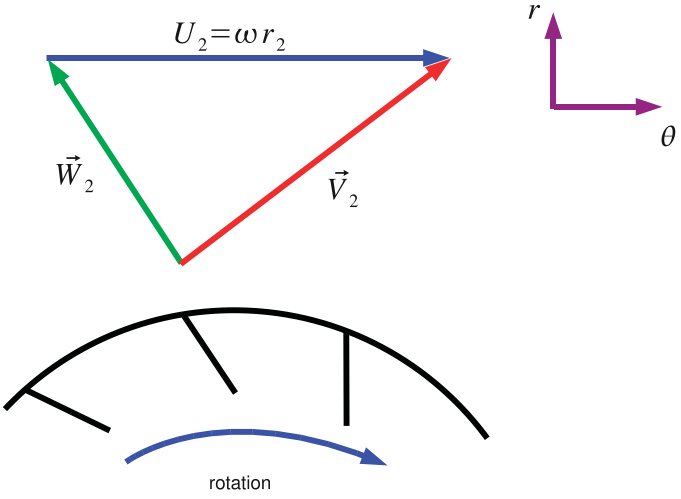


Figure 10.5: Exit from Pump Impeller

$$\omega = \frac{2\pi N}{60} = \frac{2\pi 1450}{60} = 151.8 rad/s$$

Blade Speed:

$$U_1 = \omega r_{1m} = 15.18\ m/s$$

From velocity triangle:

$$W_1^2 = V_{1x}^2 + (\omega r_{1m})^2 \Rightarrow W_1 = \sqrt{3.183^2 + 15.18^2} = 15.51\ m/s$$

$$\beta_1 = \tan^{-1}\left(\frac{W_{1\theta}}{V_{1x}}\right) = \tan^{-1}\left(\frac{-\omega r_{1m}}{V_{1x}}\right) = \tan^{-1}\left(\frac{15.18}{3.183}\right) = -78.1^\circ$$

This defines the flow at inlet completely.

Impeller Exit Sketch pump configuration and velocity triangles at exit, these are shown in Figure 10.5. Continuity is applied once again:

$$Q = 2\pi r_2 b_2 (1 - t_2) V_{r2} \Rightarrow V_{r2} = \frac{0.2}{2\pi \times 0.2 \times 0.03 \times (1 - 0.08)} = 5.77\ m/s$$

Blade Speed:

$$U_2 = \omega r_2 = 151.8 \times 0.2 = \ 30.36 m/s$$

Backward facing blades with $\beta_2 = -20^\circ$ and from the velocity triangle $U_2 + W_{2\theta} = V_{2\theta}$ but $W_{2\theta} = W_{2r} \tan \beta_2$ so:

$$V_{2\theta} = U_2 + W_{2r} \tan \beta_2$$

Now since there is no relative motion in the radial direction in the cascade view $W_{2r} = V_{2r}$ and so:

$$V_{2\theta} = 30.36 + 5.77 \tan -20^\circ = 28.26\, m/s$$

$$V_2 = \sqrt{V_{2\theta}^2 + V_{2r}^2} = \sqrt{28.26^2 + 5.77^2} = 28.84\, m/s$$

$$\alpha_2 = \tan^{-1}\left(\frac{V_{2\theta}}{V_{2r}}\right) = \tan^{-1}\left(\frac{28.26}{5.77}\right) = 78.4^\circ$$

which defines completely the exit flow from the impeller.

Specific Work Input

$$w = \omega(r_2 V_{2\theta} - r_{1m} V_{1\theta})$$

But $V_{\theta 1} = 0$ so:

$$w = 151.8 \times (0.20 \times 28.26) = 857.97\, J/kg$$

Power input is obtain directly from the specific work input:

$$P = \rho Q w = 1000 \times 0.2 \times 857.97 = 171.59\, kW$$

Head and Pump Efficiency Idea head rise is obtained from $w = g\Delta H_i$, so:

$$\Delta H_i = \frac{w}{g} = \frac{857.97}{9.81} = 87.46\, m$$

Actual Head Rise is given by $\Delta H_a = \Delta H_i - losses$ where

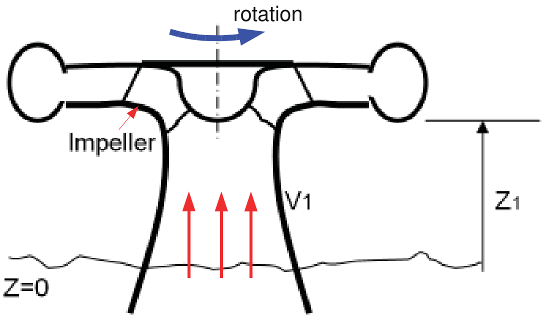
$$losses = \Delta H_{imp} + \Delta H_{diff}$$

$$\Delta H_{imp} = K_{imp} \frac{W_1^2}{2g} = 0.1 \times \frac{15.51^2}{2 \times 9.81} = 1.23\, m$$

$$K_{diff} = 1 - \eta_{diff} = 1 - 0.6 = 0.4$$

$$\Delta H_{diff} = K_{diff} \frac{V_2^2}{2g} = 0.4 \frac{28.84^2}{2 \times 9.81} = 16.96\, m$$

Therefore $\Delta H_{diff} = 16.96\, m$ - note the large diffuser loss in comparison with the impeller loss. The total head rise across the pump is therefore:



Pump inlet: 0 at atmospheric pressure
Impeller inlet: 1

Figure 10.6: Pump Inlet

$$\Delta H_a = 87.46 - 1.23 - 16.96 = 69.27\, m$$

Pump efficiency can also be calculated easily:-

$$\eta_p = \frac{\text{Ideal Work } w_i \text{ for actual } \Delta H_a}{\text{Actual Work } w}$$

Now $w_i = g\Delta H_a$ minimum work required and $w = g\Delta H_i$. This can be confusing but recall that the actual work has to supply the actual head and the losses, the ideal head is therefore always higher than the actual head produced. The efficiency is therefore given by:

$$\eta_p = \frac{g\Delta H_a}{g\Delta H_i} = \frac{69.27}{87.46} = 0.792$$

10.4 Net Positive Suction Head (NPSH)

Consider the pump shown in Figure 10.6, the pump inlet station 0 is at atmospheric pressure. The impeller inlet station 1 must be at a lower pressure p_1 to ensure flow into the pump.

For cavitation to be avoided (See Chapter 9 for details about the phenomenon of cavitation) then $p_1 > p_v$ where p_v is the saturated vapour pressure of the liquid in question. This is a property of the fluid and for water can be found in steam tables such as Rogers and Mayhew (1994). Normally this pressure is converted to a head $h = \frac{p}{\rho g}$ as pressure changes can be directly related to the physical location of the machine above or below the datum.

For satisfactory operation some margin is needed between the saturated pressure vapour and the pressure at inlet to the impeller this is called the *Net Positive Suction Head* or NPSH for short:

$$\frac{p_1 - p_v}{\rho g} = h_1 - h_v = NPSH$$

Consider the total head change from pump inlet 0 at pressure p_a and height $z = z_0 = 0$ to impeller inlet 1 where $z = z_1$:

$$H_o = H_1 + h_f$$

where h_f is the frictional head loss between station 0 and station 1. At station 0 the velocity can be regarded as very small as the reservoir that the pump is taking fluid from is generally large compared to the pipe inlet diameter and since $z_0 = 0\,m$ the total head is simply equal to the pressure head which is given by atmospheric pressure so:

$$\frac{p_a}{\rho g} = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + h_f \implies h_a = h_1 + \frac{V_1^2}{2g} + z_1 + h_f$$

this can be rearranged to give h_1:

$$h_1 = h_a - \frac{V_1^2}{2g} - z_1 - h_f$$

but $NPSH = h_1 - h_v$ so:

$$NPSH = h_a - h_v - \frac{V_1^2}{2g} - z_1 - h_f \tag{10.6}$$

This is a very important parameter in pump selection and analysis. Note how the NPSH is reduced by high inlet velocity, high height above the water surface and large friction losses in the inlet piping. In fact most pumps do not operate in the manner shown in Figure 10.6 and are usually operated below the water level, this means that z_1 would be negative increasing the NPSH.

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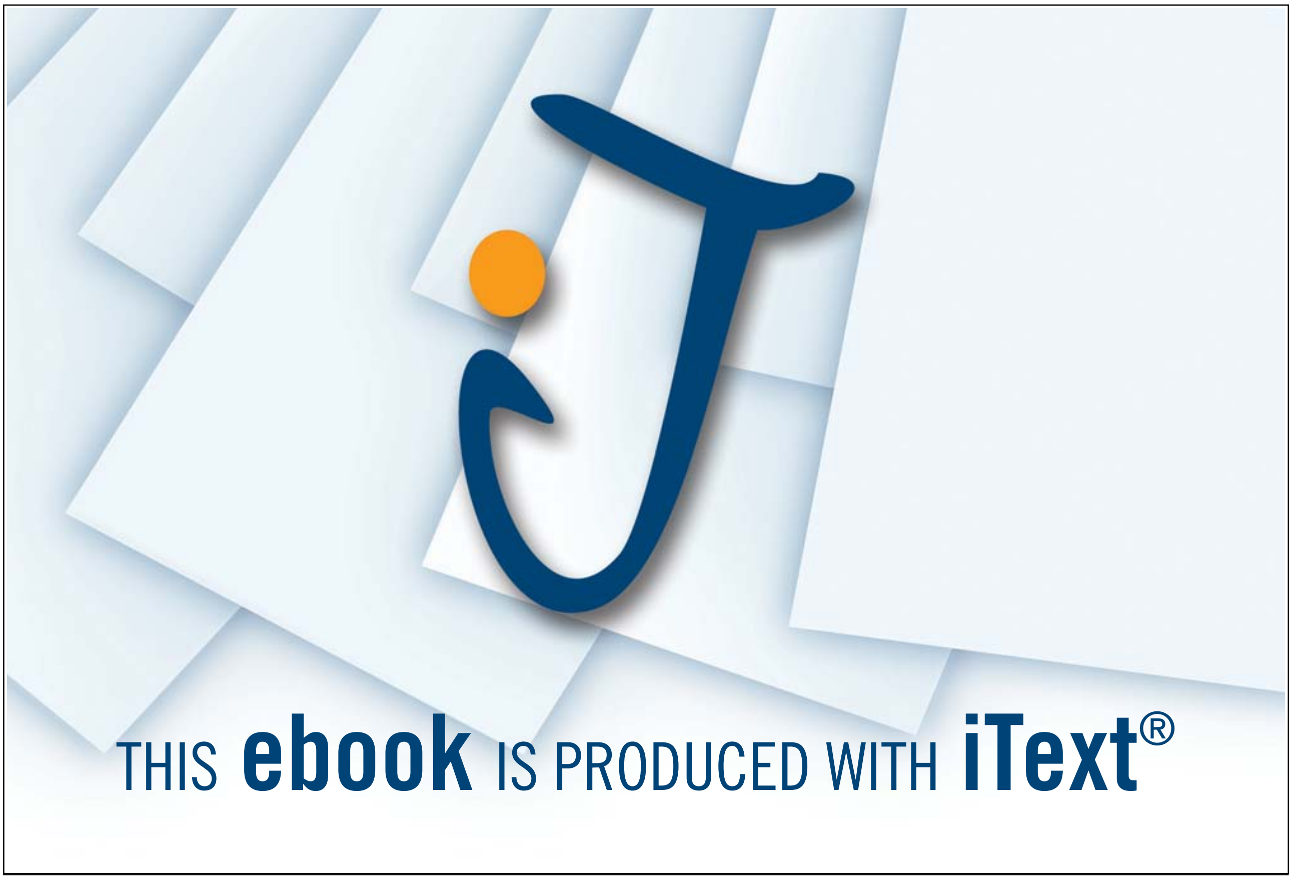

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In practise $NPSH$ values are obtained using empirical criteria. One simple treatment is to use Thoma's parameter given by:

$$\sigma = \frac{NPSH}{\Delta H} \tag{10.7}$$

where ΔH is the total head rise across the pump, cavitation is deemed to occur when $\sigma = \sigma_{crit}$ which has to obtained empirically.

10.4.1 Cavitation Example

Example Consider the previous centrifugal pump example. Calculate the impeller inlet height above the free surface water level for no cavitation. If the critical value of Thoma's parameter is $\sigma_{crit} = 0.1$. The water temperature is 25° and atmospheric pressure = 1.0 bar. The inlet duct loss is 25% of the inlet dynamic head to the pump.

Solution To determine the maximum applicable height the inlet loss is examined:

$$\Delta H_i = h_0 - \left(h_1 + \frac{V_1^2}{2g} + z_1\right) \implies z_1 = h_0 - h_1 - \frac{V_1^2}{2g} - \Delta H_i$$

The problem then reduces to finding values for $h_0, h1, V_1$ and Δh_i. The $NPSH$ can be calculated as follows:

$$NPSH = h_1 - h_v = \sigma_{crit} \times \Delta H = 0.1 \times 69.27 = 6.93\,m$$

Now $p_v = 3.17kPa$ at 25° for water, this is obtained from standard steam tables (Rogers and Mayhew (1994)). This pressure is converted to a head as follows:

$$h_v = \frac{p_v}{\rho g} = \frac{3.17 \times 10^3}{1000 \times 9.81} = 0.323\,m(abs)$$

Absolute pressures and not gauge (relative pressures) must be used. From the definition of NPSH:

$$h_1 = NPSH + h_v = 6.93 + 0.323 = 7.25\,m(abs)$$

The inlet loss is 25% of $V_1^2/2g$ so:

$$\Delta H_i = 0.25\frac{V_1^2}{2g} = 0.25\frac{3.183^2}{2 \times 9.81} = 0.129\,m$$

At the inlet to the pump (station 0) the total head is given by the inlet pressure head as $V_o, z_0 = 0$ so:

$$h_0 = \frac{p_a}{\rho g} = \frac{1.0 \times 10^5}{1000 \times 9.81} = 10.194\,m(abs)$$

So the maximum height above the water at which the pump can be placed is given by:

$$z_1 = h_0 - h_1 - \frac{V_1^2}{2g} - \Delta H_i = 10.194 - 7.25 - 0.516 - 0.129 = 2.299m$$

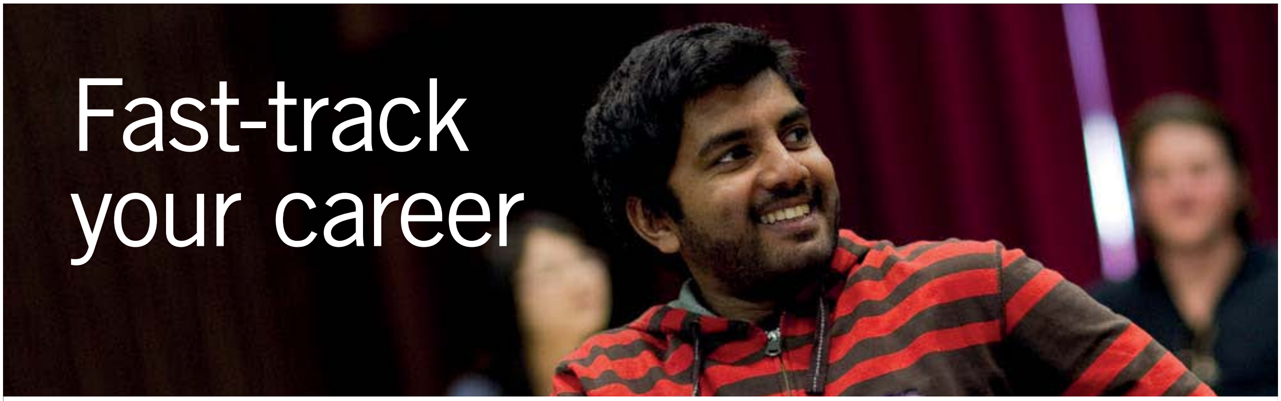
So the maximum height is around 2m above the water.

10.5 Application to Real Pumps

This book is about the basic concepts in turbomachinery and if you were to design a real pump you would need to take account of more advanced concepts such as slip - where the exit from the pump blades does not exactly follow the angle set by the blades. Although there is not space for it in this book for further details see Dixon (2005).

10.6 Problems

1. A radial flow pump delivers $0.015\ m^3/s$ when running at a speed of $1470\ rev/min$. The impeller has a uniform width of $15\,mm$, with an entry diameter of $70\,mm$ and an exit diameter of $120mm$. It has backswept blades with a relative exit flow angle of $-50°$. The blade blockage at exit is 8%. Calculate the power required to drive the pump and the relative flow angle at inlet to the impeller. *Answers:* $803\ W; -49.8°$
2. The losses for the pump in the previous question are estimated to be 15% of the relative dynamic head at inlet to the impeller and 45% of the dynamic head at inlet to the diffuser. Calculate the head delivered and the efficiency of the pump. *Answers:* $4.12\ m$; *75.4%*



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Chapter 11

Summary

There are three really difficult ideas in this book:

- *The Cascade and Meridional View.* Essential this idea is how one can relate the machine geometry to the analysis model. In the author's view this is one of the most difficult concepts in turbomachinery and is the most important idea in the book! This concept has been discussed at length throughout the book but with particular emphasis in Chapter 1.
- *Velocity Triangles.* These are essential for the calculation and understanding of machine performance and allow the graphical representation of the "key rule": $\vec{V} = \vec{U} + \vec{W}$, this concept is used throughout the book but is first explained in Chapter 2.
- *Reaction.* Very simply reaction is a choice between high work (low reaction) and high efficiency (high reaction). The geometry set by the blade angles determines velocities through the turbomachine and in Chapter 6 it was shown that this determines the reaction: the pressure drop between the stator and the rotor.

A number of key equations have been developed and applied to turbomachines:

- Continuity or conservation of mass. Equation 5.1: $\dot{m} = \rho A V$
- Conservation of angular momentum. Euler equation 5.12: $w = \omega(V_{2\theta} r_2 - V_{1\theta} r_1)$
- Conservation of energy or Rothalpy conservation across a blade row. Equation 5.17: $I = h_0 - UV_\theta = \text{ constant}$

Finally the concepts and equations have been combined to analyse the performance of a number of different machines: gas turbines, steam turbines, pumps and hydraulic turbines. This book gives an introduction to turbomachinery and (hopefully) empowers the reader to either understand more advanced texts or give an appreciation of the operation of these important devices. Further reading on turbomachinery may be found in Cumpsty (1997), Dixon (2005) or Cohen et al. (1996).

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Appendix A

Glossary of Turbomachinery Terms

Adiabatic Process A process with no heat transfer

Aerofoil An aerodynamic shape designed to produce a lift force with minimal drag (UK spelling)

Airfoil An aerodynamic shape designed to produce a lift force with minimal drag (US spelling)

Blade The name given to the part of the turbine, compressor or fan that guide the flow. Can be stationary or rotating.

Blade Passage The space between two adjacent rotor or stator blades. Essentially this is the empty space that the fluid flows through.

Blade Span The length of the blades in the radial direction

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Bucket Another name for Blade

Casing The higher radius where the case of the machine is located

Chord The shortest distance between the leading edge and the trailing edge of an aerofoil

Compressor Device that delivers energy to a fluid, usually used to describe a machine operating on a gas as the output is at a higher density to the inlet

Diaphragm Another word for stator, usually used in steam turbine practise

Enthalpy Formally: A thermodynamic property which is the sum of the internal energy of a substance and the product of the pressure and the specific volume. In more practical terms it is a measure of the energy of a fluid at a given pressure and temperature

Entropy A thermodynamic property that is constant for ideal process and increases for real processes

Fan Devices that deliver energy to a gas. Usually restricted to a device at has a modest pressure rise over the device.

Gas Turbine The name given to a complete machine including a compressor, combustor and turbine using air as a working fluid. Used to differentiate from a *Steam Turbine* which uses steam as a working fluid or as a generic name for the technology.

Guide Vane Another name for stator blades, often used with hydraulic turbines

Hub The lower radius at which the blades are attached

Hydraulic Associated with liquids, for example a hydraulic turbine is one that uses liquid (usually water) as a working fluid

Impeller Rotating element of a pump

Isentropic Process A process with constant entropy - that is a perfect process that will have the highest possible efficiency

Isothermal Process A process that takes place at constant temperature

Isobaric Process A process that takes place at constant pressure

Leading Edge The point on an aerofoil that is furthest upstream

Nozzle Another name for a stator

NPSH Net positive suction head - the margin between the saturated pressure vapour and the pressure at inlet

Pump Another name for a device that delivers energy to a liquid. The term compressor isn't appropriate as liquids are almost incompressible.

Rotor Rotating row of blades found after the stator in a turbine and before the stator for a compressor.

Runner Name for the rotating element of a machine, often used in hydraulic turbines

Scroll Casing A casing that resembles a sea shell for a radial turbomachine which aims to ensure an equal distribution of flow around the periphery by reducing the area of the device.

SFEE The steady flow energy equation a fundamental equation of Thermodynamics.

Stage A stator and a rotor combined, can be found either in a compressor or in a turbine.

Stagnation Conditions Conditions (pressure, temperature) that would occur if a fluid is brought to rest in an isentropic manner

Static Conditions Conditions (pressure, temperature etc) which occur in a fluid which is moving

Stator Stationary row of blades found before the rotor in a turbine and after the rotor for a compressor.

Station (Or Analysis Station) A convenient point in a turbomachine where analysis can take place, such as the entry to a blade row or the exit from the machine as a whole.

Steam Turbine A turbine using steam as a working fluid.

Swirl Having a tangential component

Tip The upper radius of rotor blades

Tip Clearance The gap between the moving blades and the stationary casing in a turbomachine. For high performance aeroengines this is around 1% of the blade span.

Tip Leakage The name given to the flow that goes through the tip clearance rather than following the path of the main flow through a turbomachine.

Trailing Edge The point on an aerofoil which is furthest downstream

Turbine Device for extracting energy from a fluid

Vanes Another word for blades

Vaneless Having no vanes or blades, used often to describe diffusers in pumps

Volute Another word for scroll casing

Wicket Gates Another word for guide vanes

Appendix B

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